2.31 A 6 m section of 150 Ω lossless line is driven by a source with

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \qquad \text{(V)}$$

and $Z_g = 150$ Ω. If the line, which has a relative permittivity $\varepsilon_r = 2.25$, is terminated in a load $Z_L = (150 - j50)$ Ω, determine:

(a) λ on the line.

(b) The reflection coefficient at the load.

(c) The input impedance.

(d) The input voltage $\widetilde{V}_i$.

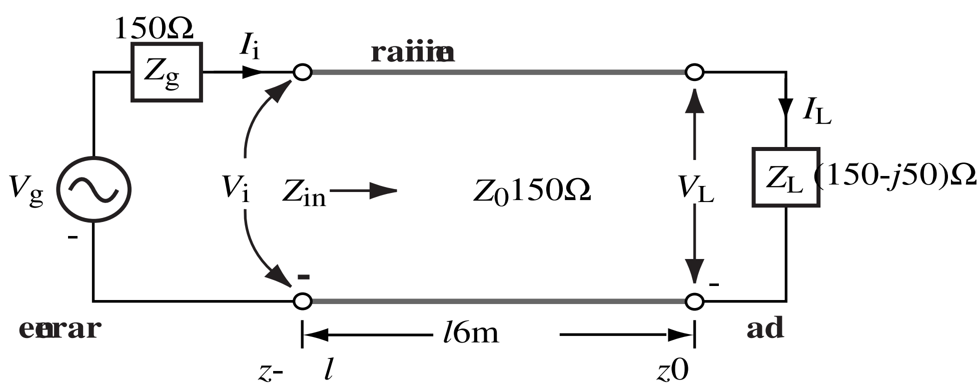
(e) The time-domain input voltage $v_i(t)$.

(f) Quantities in (a) to (d) using CD Modules 2.4 or 2.5.

Solution:

$$v_g(t) = 5\cos(8\pi \times 10^7 t - 30°) \text{ V},$$

$$\widetilde{V}_g = 5e^{-j30°} \text{ V}.$$



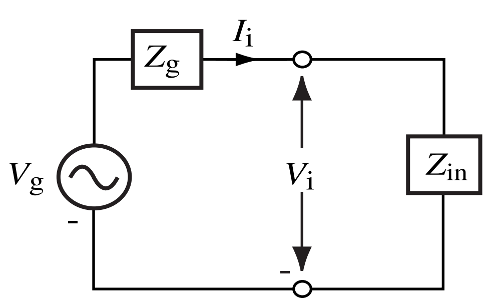


Figure P2.32: Circuit for Problem 2.32.

(a)

$$u_{\mathrm{p}} = \frac{c}{\sqrt{\varepsilon_{\mathrm{r}}}} = \frac{3\times 10^8}{\sqrt{2.25}} = 2\times 10^8 \qquad \text{(m/s)},$$

$$\lambda = \frac{u_{\mathrm{p}}}{f} = \frac{2\pi u_{\mathrm{p}}}{\omega} = \frac{2\pi\times 2\times 10^8}{8\pi\times 10^7} = 5 \ \text{ m},$$

$$\beta = \frac{\omega}{u_{\mathrm{p}}} = \frac{8\pi\times 10^7}{2\times 10^8} = 0.4\pi \qquad \text{(rad/m)},$$

$$\beta l = 0.4\pi\times 6 = 2.4\pi \qquad \text{(rad)}.$$

Since this exceeds 2π (rad), we can subtract 2π, which leaves a remainder $\beta l = 0.4\pi$ (rad).

(b) $\Gamma = \dfrac{Z_{\mathrm{L}}-Z_0}{Z_{\mathrm{L}}+Z_0} = \dfrac{150-j50-150}{150-j50+150} = \dfrac{-j50}{300-j50} = 0.16\,e^{-j80.54^\circ}$.

(c)

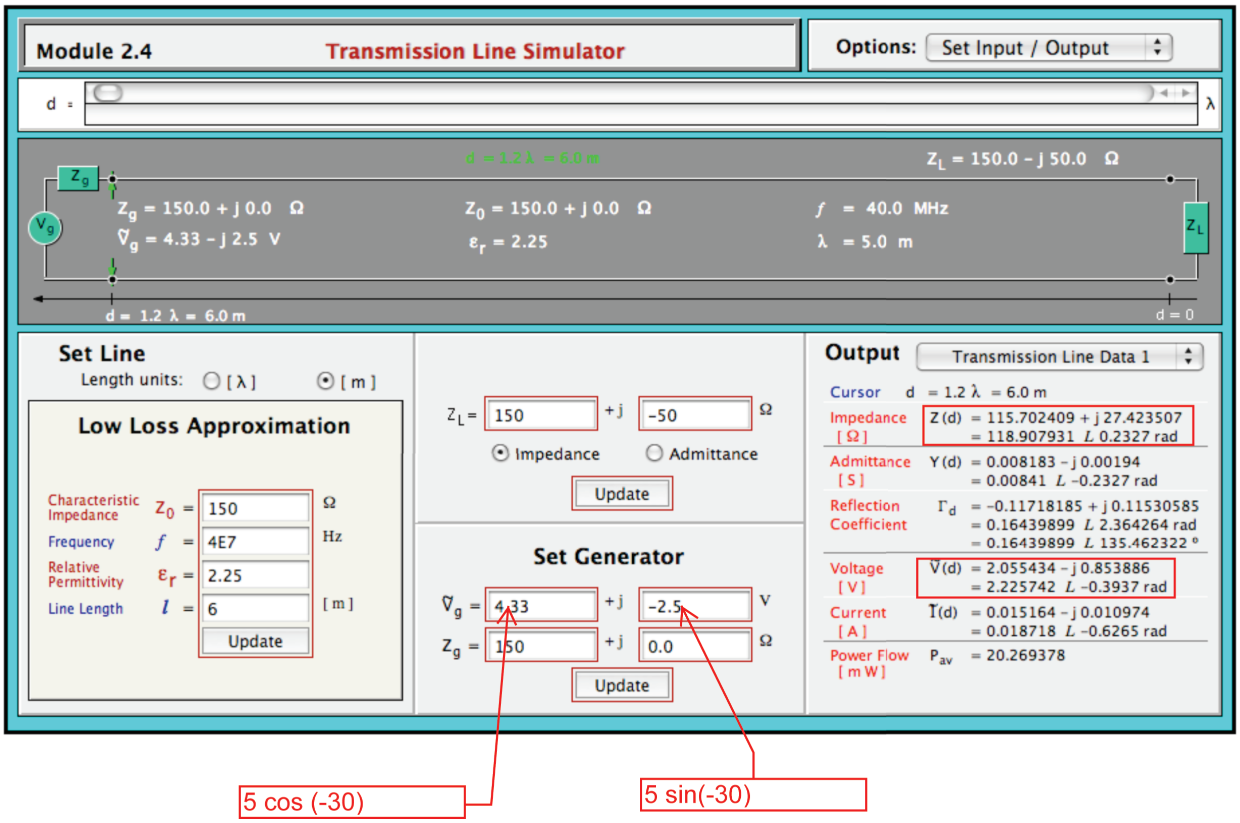
$$Z_{\mathrm{in}} = Z_0\left[\frac{Z_{\mathrm{L}}+jZ_0\tan\beta l}{Z_0+jZ_L\tan\beta l}\right] = 150\left[\frac{(150-j50)+j150\tan(0.4\pi)}{150+j(150-j50)\tan(0.4\pi)}\right] = (115.70+j27.42)\ \Omega.$$

(d)

$$\widetilde{V}_{\mathrm{i}} = \frac{\widetilde{V}_{\mathrm{g}}Z_{\mathrm{in}}}{Z_{\mathrm{g}}+Z_{\mathrm{in}}} = \frac{5e^{-j30^\circ}(115.7+j27.42)}{150+115.7+j27.42} = 5e^{-j30^\circ}\left(\frac{115.7+j27.42}{265.7+j27.42}\right) = 5e^{-j30^\circ}\times 0.44\,e^{j7.44^\circ} = 2.2\,e^{-j22.56^\circ} \qquad \text{(V)}.$$

(e)

$$v_{\mathrm{i}}(t) = \mathfrak{Re}[\widetilde{V}_{\mathrm{i}}e^{j\omega t}] = \mathfrak{Re}[2.2\,e^{-j22.56^\circ}e^{j\omega t}] = 2.2\cos(8\pi\times 10^7 t - 22.56^\circ)\ \text{ V}.$$


Module 2.4
Transmission Line Simulator
Options: Set Input / Output
d =
λ
d = 1.2 λ = 6.0 m
Z_L = 150.0 – j 50.0 Ω
Z_g
V_g
Z_g = 150.0 + j 0.0 Ω
$\tilde{V}_g$ = 4.33 – j 2.5 V
Z_0 = 150.0 + j 0.0 Ω
ε_r = 2.25
f = 40.0 MHz
λ = 5.0 m
Z_L
d = 1.2 λ = 6.0 m
d = 0
Set Line
Length units: [λ] [m]
Low Loss Approximation
Characteristic Impedance Z_0 = 150 Ω
Frequency f = 4E7 Hz
Relative Permittivity ε_r = 2.25
Line Length l = 6 [m]
Update
Z_L = 150 + j -50 Ω
Impedance
Admittance
Update
Set Generator
$\tilde{V}_g$ = 4.33 + j -2.5 V
Z_g = 150 + j 0.0 Ω
Update
Output
Transmission Line Data 1
Cursor d = 1.2 λ = 6.0 m
Impedance [Ω] Z (d) = 115.702409 + j 27.423507
= 118.907931 ∠ 0.2327 rad
Admittance [S] Y (d) = 0.008183 – j 0.00194
= 0.00841 ∠ -0.2327 rad
Reflection Coefficient Γ_d = -0.11718185 + j 0.11530585
= 0.16439899 ∠ 2.364264 rad
= 0.16439899 ∠ 135.462322 °
Voltage [V] $\tilde{V}$(d) = 2.055434 – j 0.853886
= 2.225742 ∠ -0.3937 rad
Current [A] $\tilde{I}$(d) = 0.015164 – j 0.010974
= 0.018718 ∠ -0.6265 rad
Power Flow [m W] P_{av} = 20.269378
5 cos (-30)
5 sin(-30)