

2.32 A voltage generator with

$$v_g(t) = 5 \cos(2\pi \times 10^9 t) \text{ V}$$

and internal impedance $Z_g = 50 \Omega$ is connected to a 50Ω lossless air-spaced transmission line. The line length is 5 cm and the line is terminated in a load with impedance $Z_L = (100 - j100) \Omega$. Determine:

- (a) Γ at the load.
- (b) Z_{in} at the input to the transmission line.
- (c) The input voltage \tilde{V}_i and input current \tilde{I}_i .
- (d) The quantities in (a)–(c) using CD Modules 2.4 or 2.5.

Solution:

- (a) From Eq. (2.59),

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{(100 - j100) - 50}{(100 - j100) + 50} = 0.62 \exp -j29.7^\circ.$$

(b) All formulae for Z_{in} require knowledge of $\beta = \omega/u_p$. Since the line is an air line, $u_p = c$, and from the expression for $v_g(t)$ we conclude $\omega = 2\pi \times 10^9$ rad/s. Therefore

$$\beta = \frac{2\pi \times 10^9 \text{ rad/s}}{3 \times 10^8 \text{ m/s}} = \frac{20\pi}{3} \text{ rad/m}.$$

Then, using Eq. (2.79),

$$\begin{aligned} Z_{in} &= Z_0 \left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right) \\ &= 50 \left[\frac{(100 - j100) + j50 \tan \left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm} \right)}{50 + j(100 - j100) \tan \left(\frac{20\pi}{3} \text{ rad/m} \times 5 \text{ cm} \right)} \right] \\ &= 50 \left[\frac{(100 - j100) + j50 \tan \left(\frac{\pi}{3} \text{ rad} \right)}{50 + j(100 - j100) \tan \left(\frac{\pi}{3} \text{ rad} \right)} \right] = (12.5 - j12.7) \Omega. \end{aligned}$$

- (c) In phasor domain, $\tilde{V}_g = 5 \text{ V} \exp j0^\circ$. From Eq. (2.80),

$$\tilde{V}_i = \frac{\tilde{V}_g Z_{in}}{Z_g + Z_{in}} = \frac{5 \times (12.5 - j12.7)}{50 + (12.5 - j12.7)} = 1.40 \exp -j34.0^\circ \quad (\text{V}),$$

and also from Eq. (2.80),

$$\tilde{I}_i = \frac{\tilde{V}_i}{Z_{in}} = \frac{1.4 \exp -j34.0^\circ}{(12.5 - j12.7)} = 78.4 \exp j11.5^\circ \quad (\text{mA}).$$

Module 2.4

Transmission Line Simulator

Options: Set Input / Output

d =

λ

$d = 0.1666 \lambda = 49.98 \text{ mm}$
 $Z_L = 100.0 - j 100.0 \ \Omega$

$Z_g = 50.0 + j 0.0 \ \Omega$
 $Z_0 = 50.0 + j 0.0 \ \Omega$
 $f = 1.0 \text{ GHz}$

$V_g = 5.0 + j 0.0 \text{ V}$
 $\epsilon_r = 1.0$
 $\lambda = 300.0 \text{ mm}$

$d = 0.166667 \lambda = 50.0 \text{ mm}$
 $d = 0$

Set Line

Length units: ☐ [λ] ☒ [m]

Low Loss Approximation

Characteristic Impedance $Z_0 =$ Ω

Frequency $f =$ Hz

Relative Permittivity $\epsilon_r =$

Line Length $l =$ [m]

Update

$Z_L =$ $+j$ Ω

☒ Impedance
 ☐ Admittance

Update

Set Generator

$V_g =$ $+j$ V
 $Z_g =$ $+j$ Ω

Update

Output

Transmission Line Data 1

Cursor $d = 0.1666 \lambda = 49.98 \text{ mm}$

Impedance $Z(d) = 12.530782 - j 12.743838$
[Ω]
 $= 17.87249 \angle -0.7938 \text{ rad}$

Admittance $Y(d) = 0.039229 + j 0.039896$
[S]
 $= 0.055952 \angle 0.7938 \text{ rad}$

Reflection Coefficient $\Gamma_d = -0.53543815 - j 0.31292389$
 $= 0.62017367 \angle -2.612703 \text{ rad}$
 $= 0.62017367 \angle -149.696881^\circ$

Voltage $V(d) = 1.161077 - j 0.782796$
[V]
 $= 1.40031 \angle -0.5932 \text{ rad}$

Current $I(d) = 0.076778 + j 0.015614$
[A]
 $= 0.07835 \angle 0.2006 \text{ rad}$

Power Flow $P_{av} = 38.461538$
[mW]