

2.37 At an operating frequency of 300 MHz, it is desired to use a section of a lossless $50\ \Omega$ transmission line terminated in a short circuit to construct an equivalent load with reactance $X = 40\ \Omega$. If the phase velocity of the line is $0.75c$, what is the shortest possible line length that would exhibit the desired reactance at its input? Verify your results using CD Module 2.5.

Solution:

$$\beta = \omega/u_p = \frac{(2\pi\text{ rad/cycle}) \times (300 \times 10^6\text{ cycle/s})}{0.75 \times (3 \times 10^8\text{ m/s})} = 8.38\text{ rad/m}.$$

On a lossless short-circuited transmission line, the input impedance is always purely imaginary; i.e., $Z_{\text{in}}^{\text{sc}} = jX_{\text{in}}^{\text{sc}}$. Solving Eq. (2.84) for the line length,

$$l = \frac{1}{\beta} \tan^{-1} \left(\frac{X_{\text{in}}^{\text{sc}}}{Z_0} \right) = \frac{1}{8.38\text{ rad/m}} \tan^{-1} \left(\frac{40\ \Omega}{50\ \Omega} \right) = \frac{(0.675 + n\pi)\text{ rad}}{8.38\text{ rad/m}},$$

for which the smallest positive solution is 8.05 cm (with $n = 0$). Since $u_p = 0.75c$,

$$\epsilon_{\text{eff}} = \left(\frac{c}{u_p} \right)^2 = 1.777.$$

From Module 2.5, $Z(d) = j40\ \Omega$ when

$$d = 0.107388\lambda.$$

But

$$\lambda = \frac{u_p}{f} = \frac{0.75 \times 3 \times 10^8}{3 \times 10^8} = 0.75\text{ m}.$$

Hence,

$$d = 0.107388 \times 0.75 = 0.0805\text{ m} = 8.05\text{ cm}.$$

Options: Set Line and Load

$$Z_L = 0 \quad (\text{Short Circuit})$$

$$Z_0 = 50.0 \, \Omega$$

$$\varepsilon_f = 1.777777$$
$$d = 0$$

frequency

(press Update to activate choice)

Set Load

$$Z_0 = 50.0 \quad [\Omega]$$
$$\epsilon_r = 1.777777$$
$$l = 0.5 \text{ [m]}$$

Update

$$Z_L = 0 + 0 \text{ } [\Omega]$$

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