2.44 For the circuit shown in Fig. P2.44, calculate the average incident power, the average reflected power, and the average power transmitted into the infinite 100-Ω line. The $\lambda/2$ line is lossless and the infinitely long line is slightly lossy. (Hint: The input impedance of an infinitely long line is equal to its characteristic impedance so long as $\alpha \neq 0$.)

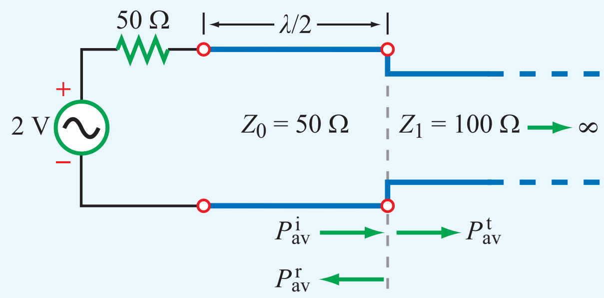

Figure P2.44 Circuit for Problem 2.44.

Solution: Considering the semi-infinite transmission line as equivalent to a load (since all power sent down the line is lost to the rest of the circuit), $Z_L = Z_1 = 100\ \Omega$. Since the feed line is $\lambda/2$ in length, Eq. (2.96) gives $Z_{in} = Z_L = 100\ \Omega$ and $\beta l = (2\pi/\lambda)(\lambda/2) = \pi$, so $\exp \pm j\beta l = -1$. Hence

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 - 50}{100 + 50} = \frac{1}{3}.$$

Also, converting the generator to a phasor gives $\widetilde{V}_g = 2 \exp j0^\circ$ (V). Plugging all these results into Eq. (2.82),

$$V_0^+ = \left(\frac{\widetilde{V}_g Z_{in}}{Z_g + Z_{in}}\right)\left(\frac{1}{\exp j\beta l + \Gamma \exp -j\beta l}\right) = \left(\frac{2 \times 100}{50 + 100}\right)\left[\frac{1}{(-1) + \frac{1}{3}(-1)}\right]$$
$$= 1 \exp j180^\circ = -1 \qquad \text{(V)}.$$

From Eqs. (2.104), (2.105), and (2.106),

$$P_{av}^i = \frac{|V_0^+|^2}{2Z_0} = \frac{|1 \exp j180^\circ|^2}{2 \times 50} = 10.0 \text{ mW},$$

$$P_{av}^r = -|\Gamma|^2 P_{av}^i = -\left|\frac{1}{3}\right|^2 \times 10 \text{ mW} = -1.1 \text{ mW},$$

$$P_{av} = P_{av}^t = P_{av}^i + P_{av}^r = 10.0 \text{ mW} - 1.1 \text{ mW} = 8.9 \text{ mW}.$$