2.78 In response to a step voltage, the voltage waveform shown in Fig. P2.78 was observed at the sending end of a shorted line with $Z_0 = 50\ \Omega$ and $\varepsilon_r = 2.25$. Determine V_g, R_g, and the line length.

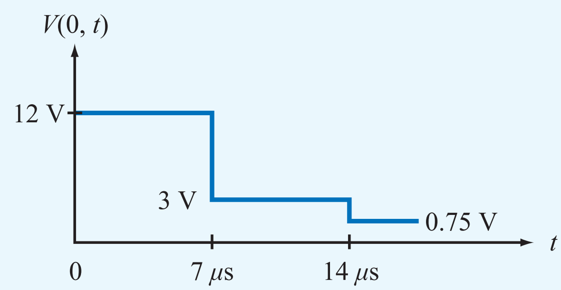

Figure P2.78 Voltage waveform of Problem 2.78.

Solution:

$$u_p = \frac{c}{\sqrt{\varepsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.25}} = 2 \times 10^8 \text{ m/s},$$

$$7\ \mu\text{s} = 7 \times 10^{-6} \text{ s} = \frac{2l}{u_p} = \frac{2l}{2 \times 10^8}.$$

Hence, $l = 700$ m.

From the voltage waveform, $V_1^+ = 12$ V. At $t = 7\mu$s, the voltage at the sending end is

$$V(z=0, t=7\mu\text{s}) = V_1^+ + \Gamma_L V_1^+ + \Gamma_g \Gamma_L V_1^+ = -\Gamma_g V_1^+ \qquad \text{(because } \Gamma_L = -1\text{)}.$$

Hence, 3 V$= -\Gamma_g \times 12$ V, or $\Gamma_g = -0.25$. From Eq. (2.153),

$$R_g = Z_0 \left(\frac{1 + \Gamma_g}{1 - \Gamma_g} \right) = 50 \left(\frac{1 - 0.25}{1 + 0.25} \right) = 30\ \Omega.$$

Also,

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0}, \qquad \text{or} \quad 12 = \frac{V_g \times 50}{30 + 50},$$

which gives $V_g = 19.2$ V.