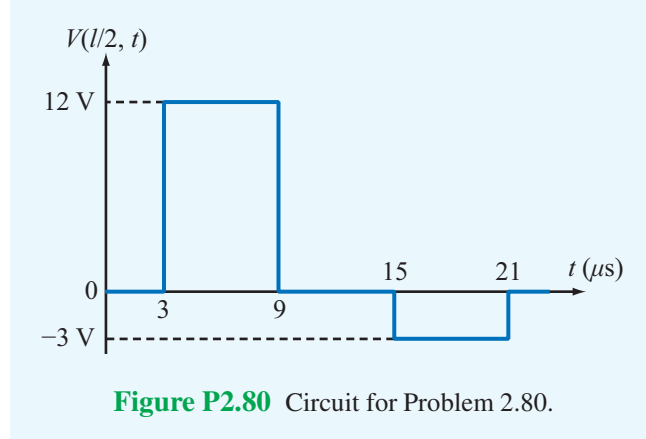


2.80 In response to a step voltage, the voltage waveform shown in Fig. P2.80 was observed at the midpoint of a lossless transmission line with $Z_0 = 50 \, \Omega$ and $u_p = 1 \times 10^8 \, \text{m/s}$. Determine: (a) the length of the line, (b) Z_L , (c) R_g , and (d) V_g .



Solution:

(a) Since it takes $3 \, \mu\text{s}$ to reach the middle of the line, the line length must be

$$l = 2(3 \times 10^{-6} \times u_p) = 2 \times 3 \times 10^{-6} \times 1 \times 10^8 = 600 \, \text{m}.$$

(b) From the voltage waveform shown in the figure, the duration of the first rectangle is $6 \, \mu\text{s}$, representing the time it takes the incident voltage V_1^+ to travel from the midpoint of the line to the load and back. The fact that the voltage drops to zero at $t = 9 \, \mu\text{s}$ implies that the reflected wave is exactly equal to V_1^+ in magnitude, but opposite in polarity. That is,

$$V_1^- = -V_1^+.$$

This in turn implies that $\Gamma_L = -1$, which means that the load is a short circuit:

$$Z_L = 0.$$

(c) After V_1^- arrives at the generator end, it encounters a reflection coefficient Γ_g . The voltage at $15 \, \mu\text{s}$ is composed of:

$$\begin{aligned} V &= V_1^+ + V_1^- + V_2^+ \\ &= (1 + \Gamma_L + \Gamma_L \Gamma_g) V_1^+ \\ \frac{V}{V_1^+} &= 1 - 1 - \Gamma_g \end{aligned}$$

From the figure, $V/V_1^+ = -3/12 = -1/4$. Hence,

$$\Gamma_g = \frac{1}{4},$$

which means that

$$R_g = \left(\frac{1 + \Gamma_g}{1 - \Gamma_g} \right) Z_0 = \left(\frac{1 + 0.25}{1 - 0.25} \right) 50 = 83.3 \, \Omega.$$

(d)

$$V_1^+ = 12 = \frac{V_g Z_0}{R_g + Z_0}$$

$$V_g = \frac{12(R_g + Z_0)}{Z_0} = \frac{12(83.3 + 50)}{50} = 32 \, \text{V}.$$