3.13 Two lines in the x–y plane are described by the expressions:

$$\begin{array}{ll} \text{Line 1} & x+2y=-6, \\ \text{Line 2} & 3x+4y=8. \end{array}$$

Use vector algebra to find the smaller angle between the lines at their intersection point.

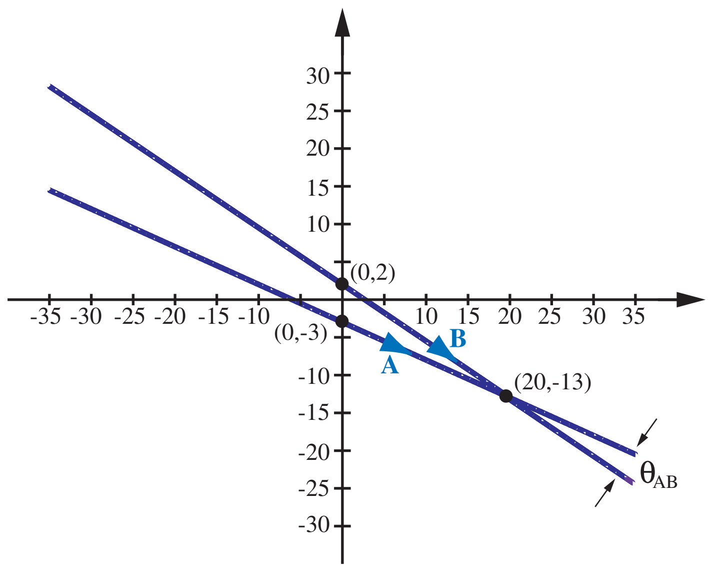

Figure P3.13 Lines 1 and 2.

Solution: Intersection point is found by solving the two equations simultaneously:

$$-2x-4y=12,$$
$$3x+4y=8.$$

The sum gives $x=20$, which, when used in the first equation, gives $y=-13$.
Hence, intersection point is $(20,-13)$.
Another point on line 1 is $x=0,\ \ y=-3$. Vector $\mathbf{A}$ from $(0,-3)$ to $(20,-13)$ is

$$\mathbf{A}=\hat{\mathbf{x}}(20)+\hat{\mathbf{y}}(-13+3)=\hat{\mathbf{x}}20-\hat{\mathbf{y}}10,$$
$$|\mathbf{A}|=\sqrt{20^2+10^2}=\sqrt{500}.$$

A point on line 2 is $x=0,\ \ y=2$. Vector $\mathbf{B}$ from $(0,2)$ to $(20,-13)$ is

$$\mathbf{B}=\hat{\mathbf{x}}(20)+\hat{\mathbf{y}}(-13-2)=\hat{\mathbf{x}}20-\hat{\mathbf{y}}15,$$

$$|\mathbf{B}| = \sqrt{20^2 + 15^2} = \sqrt{625}.$$

Angle between **A** and **B** is

$$\theta_{AB} = \cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{|\mathbf{A}||\mathbf{B}|}\right) = \cos^{-1}\left(\frac{400+150}{\sqrt{500}\cdot\sqrt{625}}\right) = 10.3^\circ.$$