3.14 Show that, given two vectors **A** and **B**,

(a) the vector **C** defined as the vector component of **B** in the direction of **A** is given by

$$\mathbf{C} = \hat{\mathbf{a}}(\mathbf{B}\cdot\hat{\mathbf{a}}) = \frac{\mathbf{A}(\mathbf{B}\cdot\mathbf{A})}{|\mathbf{A}|^2},$$

where $\hat{\mathbf{a}}$ is the unit vector of **A**, and

(b) the vector **D** defined as the vector component of **B** perpendicular to **A** is given by

$$\mathbf{D} = \mathbf{B} - \frac{\mathbf{A}(\mathbf{B}\cdot\mathbf{A})}{|\mathbf{A}|^2}.$$

Solution:

(a) By definition, $\mathbf{B}\cdot\hat{\mathbf{a}}$ is the component of **B** along $\hat{\mathbf{a}}$. The vector component of $(\mathbf{B}\cdot\hat{\mathbf{a}})$ along **A** is

$$\mathbf{C} = \hat{\mathbf{a}}(\mathbf{B}\cdot\hat{\mathbf{a}}) = \frac{\mathbf{A}}{|\mathbf{A}|}\left(\mathbf{B}\cdot\frac{\mathbf{A}}{|\mathbf{A}|}\right) = \frac{\mathbf{A}(\mathbf{B}\cdot\mathbf{A})}{|\mathbf{A}|^2}.$$

(b) The figure shows vectors **A**, **B**, and **C**, where **C** is the projection of **B** along **A**. It is clear from the triangle that

$$\mathbf{B} = \mathbf{C} + \mathbf{D},$$

or

$$\mathbf{D} = \mathbf{B} - \mathbf{C} = \mathbf{B} - \frac{\mathbf{A}(\mathbf{B}\cdot\mathbf{A})}{|\mathbf{A}|^2}.$$

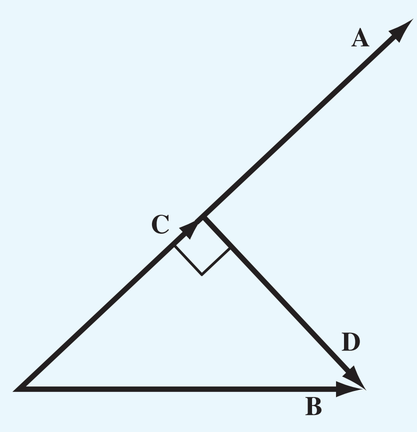

Figure P3.14 Relationships between vectors **A**, **B**, **C**, and **D**