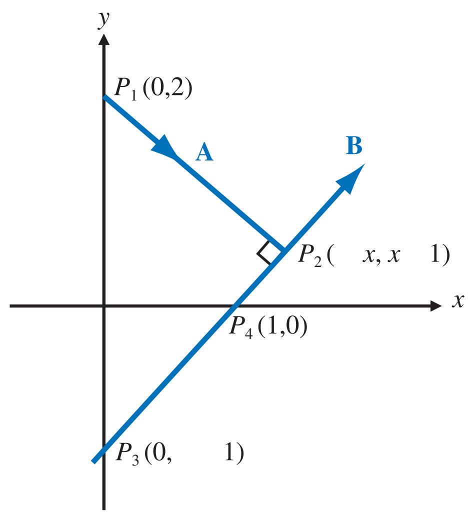
3.18 A given line is described by the equation:

$$y = x - 1.$$

Vector **A** starts at point $P_1 = (0,2)$ and ends at point P_2 on the line such that **A** is orthogonal to the line. Find an expression for **A**.

Solution: We first plot the given line.



Next we find a vector **B** which connects point $P_3 = (0,1)$ to point $P_4 = (1,0)$, both of which are on the line. Hence,

$$\mathbf{B} = \hat{\mathbf{x}}\,(1-0) + \hat{\mathbf{y}}\,(0+1) = \hat{\mathbf{x}} + \hat{\mathbf{y}}\,.$$

Vector **A** starts at $P_1 = (0,2)$ and ends on the line at P_2. If the x-coordinate of P_2 is x, then its y-coordinate has to be $y = x - 1$, per the equation for the line. Thus, P_2 is at $(x, x-1)$, and vector **A** is

$$\mathbf{A} = \hat{\mathbf{x}}\,(x-0) + \hat{\mathbf{y}}\,(x-1-2) = \hat{\mathbf{x}}x + \hat{\mathbf{y}}\,(x-3).$$

Since **A** is orthogonal to **B**,

$$\begin{aligned}
\mathbf{A}\cdot\mathbf{B} &= 0, \\
[\hat{\mathbf{x}}x + \hat{\mathbf{y}}\,(x-3)]\cdot(\hat{\mathbf{x}} + \hat{\mathbf{y}}) &= 0 \\
x + x - 3 &= 0 \\
x &= \frac{3}{2}\,.
\end{aligned}$$

Finally,

$$\begin{aligned}\mathbf{A} = \hat{\mathbf{x}}x + \hat{\mathbf{y}}(x-3) &= \hat{\mathbf{x}}\frac{3}{2} + \hat{\mathbf{y}}\left(\frac{3}{2} - 3\right) \\ &= \hat{\mathbf{x}}\frac{3}{2} - \hat{\mathbf{y}}\frac{3}{2}\,.\end{aligned}$$