

3.27 Find the volumes described by

(a) $2 \leq r \leq 5$; $\pi/2 \leq \phi \leq \pi$; $0 \leq z \leq 2$,

(b) $0 \leq R \leq 5$; $0 \leq \theta \leq \pi/3$; $0 \leq \phi \leq 2\pi$.

Also sketch the outline of each volume.

Solution:

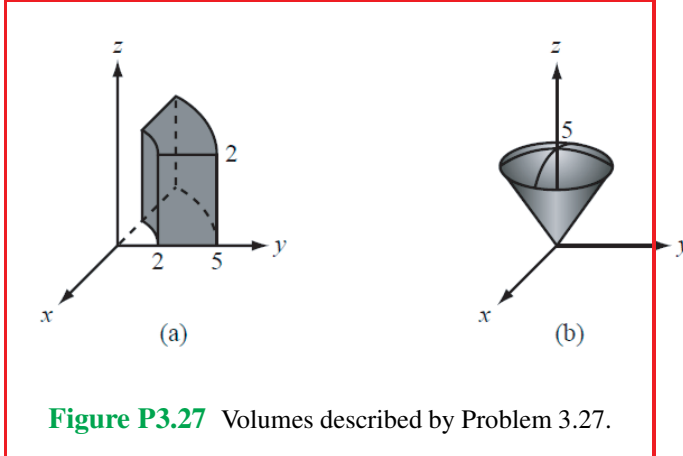


Figure P3.27 Volumes described by Problem 3.27.

(a) From Eq. (3.44),

$$V = \int_{z=0}^2 \int_{\phi=\pi/2}^{\pi} \int_{r=2}^5 r dr d\phi dz = \left(\left(\left(\frac{1}{2} r^2 \phi z \right) \Big|_{r=2}^5 \right) \Big|_{\phi=\pi/2}^{\pi} \right) \Big|_{z=0}^2 = \frac{21\pi}{2}.$$

(b) From Eq. (3.50e),

$$\begin{aligned} V &= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/3} \int_{R=0}^5 R^2 \sin \theta dR d\theta d\phi \\ &= \left(\left(\left(-\cos \theta \frac{R^3}{3} \phi \right) \Big|_{R=0}^5 \right) \Big|_{\theta=0}^{\pi/3} \right) \Big|_{\phi=0}^{2\pi} = \frac{125\pi}{3}. \end{aligned}$$