

3.28 A vector field is given in cylindrical coordinates by

$$\mathbf{E} = \hat{\mathbf{r}}r \cos \phi + \hat{\boldsymbol{\phi}}r \sin \phi + \hat{\mathbf{z}}z^3.$$

Point $P = (2, \pi, 3)$ is located on the surface of the cylinder described by $r = 2$. At point P , find:

- (a) the vector component of \mathbf{E} perpendicular to the cylinder,
- (b) the vector component of \mathbf{E} tangential to the cylinder.

Solution:

(a) $\mathbf{E}_n = \hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{E}) = \hat{\mathbf{r}}[\hat{\mathbf{r}} \cdot (\hat{\mathbf{r}}r \cos \phi + \hat{\boldsymbol{\phi}}r \sin \phi + \hat{\mathbf{z}}z^3)] = \hat{\mathbf{r}}r \cos \phi.$

At $P = (2, \pi, 3)$, $\mathbf{E}_n = \hat{\mathbf{r}}2 \cos \pi = -\hat{\mathbf{r}}2.$

(b) $\mathbf{E}_t = \mathbf{E} - \mathbf{E}_n = \hat{\boldsymbol{\phi}}r \sin \phi + \hat{\mathbf{z}}z^3.$

At $P = (2, \pi, 3)$, $\mathbf{E}_t = \hat{\boldsymbol{\phi}}2 \sin \pi + \hat{\mathbf{z}}3^3 = \hat{\mathbf{z}}27.$
