**3.29** At a given point in space, vectors **A** and **B** are given in spherical coordinates by

$$\mathbf{A} = \hat{\mathbf{R}}4 + \hat{\boldsymbol{\theta}}2 - \hat{\boldsymbol{\phi}},$$
$$\mathbf{B} = -\hat{\mathbf{R}}2 + \hat{\boldsymbol{\phi}}3.$$

Find:

**(a)** the scalar component, or projection, of **B** in the direction of **A**,

**(b)** the vector component of **B** in the direction of **A**,

**(c)** the vector component of **B** perpendicular to **A**.

**Solution:**

**(a)** Scalar component of **B** in direction of **A**:

$$C = \mathbf{B}\cdot\hat{\mathbf{a}} = \mathbf{B}\cdot\frac{\mathbf{A}}{|\mathbf{A}|} = (-\hat{\mathbf{R}}2 + \hat{\boldsymbol{\phi}}3)\cdot\frac{(\hat{\mathbf{R}}4 + \hat{\boldsymbol{\theta}}2 - \hat{\boldsymbol{\phi}})}{\sqrt{16+4+1}}$$
$$= \frac{-8-3}{\sqrt{21}} = -\frac{11}{\sqrt{21}} = -2.4.$$

**(b)** Vector component of **B** in direction of **A**:

$$\mathbf{C} = \hat{\mathbf{a}}C = \mathbf{A}\frac{C}{|\mathbf{A}|} = (\hat{\mathbf{R}}4 + \hat{\boldsymbol{\theta}}2 - \hat{\boldsymbol{\phi}})\frac{(-2.4)}{\sqrt{21}}$$
$$= -(\hat{\mathbf{R}}2.09 + \hat{\boldsymbol{\theta}}1.05 - \hat{\boldsymbol{\phi}}0.52).$$

**(c)** Vector component of **B** perpendicular to **A**:

$$\mathbf{D} = \mathbf{B} - \mathbf{C} = (-\hat{\mathbf{R}}2 + \hat{\boldsymbol{\phi}}3) + (\hat{\mathbf{R}}2.09 + \hat{\boldsymbol{\theta}}1.05 - \hat{\boldsymbol{\phi}}0.52)$$
$$= \hat{\mathbf{R}}0.09 + \hat{\boldsymbol{\theta}}1.05 + \hat{\boldsymbol{\phi}}2.48.$$