3.34 Transform the following vectors into cylindrical coordinates and then evaluate them at the indicated points:

(a) $\mathbf{A} = \hat{\mathbf{x}}(x+y)$ at $P_1 = (1,2,3)$,

(b) $\mathbf{B} = \hat{\mathbf{x}}(y-x) + \hat{\mathbf{y}}(x-y)$ at $P_2 = (1,0,2)$,

(c) $\mathbf{C} = \hat{\mathbf{x}}y^2/(x^2+y^2) - \hat{\mathbf{y}}x^2/(x^2+y^2) + \hat{\mathbf{z}}4$ at $P_3 = (1,-1,2)$,

(d) $\mathbf{D} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta + \hat{\boldsymbol{\phi}}\cos^2\phi$ at $P_4(2,\pi/2,\pi/4)$,

(e) $\mathbf{E} = \hat{\mathbf{R}}\cos\phi + \hat{\boldsymbol{\theta}}\sin\phi + \hat{\boldsymbol{\phi}}\sin^2\theta$ at $P_5 = (3,\pi/2,\pi)$.

Solution: From Table 3-2:

(a)

$$\begin{aligned}
\vec{A} &= \left(\hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi\right)(r\cos\phi + r\sin\phi) \\
&= \hat{\mathbf{r}}r\cos\phi(\cos\phi+\sin\phi) - \hat{\boldsymbol{\phi}}r\sin\phi(\cos\phi+\sin\phi), \\
P_1 &= \left(\sqrt{1^2+2^2}, \tan^{-1}(2/1), 3\right) = \left(\sqrt{5}, 63.4^\circ, 3\right), \\
\vec{A}(P_1) &= \left(\hat{\mathbf{r}}0.447 - \hat{\boldsymbol{\phi}}0.894\right)\sqrt{5}\,(.447+.894) = \hat{\mathbf{r}}1.34 - \hat{\boldsymbol{\phi}}2.68.
\end{aligned}$$

(b)

$$\begin{aligned}
\vec{B} &= \left(\hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi\right)(r\sin\phi - r\cos\phi) + \left(\hat{\boldsymbol{\phi}}\cos\phi + \hat{\mathbf{r}}\sin\phi\right)(r\cos\phi - r\sin\phi) \\
&= \hat{\mathbf{r}}r(2\sin\phi\cos\phi - 1) + \hat{\boldsymbol{\phi}}r\left(\cos^2\phi - \sin^2\phi\right) = \hat{\mathbf{r}}r(\sin 2\phi - 1) + \hat{\boldsymbol{\phi}}r\cos 2\phi, \\
P_2 &= \left(\sqrt{1^2+0^2}, \tan^{-1}(0/1), 2\right) = (1, 0^\circ, 2), \\
\vec{B}(P_2) &= -\hat{\mathbf{r}} + \hat{\boldsymbol{\phi}}.
\end{aligned}$$

(c)

$$\begin{aligned}
\vec{C} &= \left(\hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi\right)\frac{r^2\sin^2\phi}{r^2} - \left(\hat{\boldsymbol{\phi}}\cos\phi + \hat{\mathbf{r}}\sin\phi\right)\frac{r^2\cos^2\phi}{r^2} + \hat{\mathbf{z}}4 \\
&= \hat{\mathbf{r}}\sin\phi\cos\phi(\sin\phi - \cos\phi) - \hat{\boldsymbol{\phi}}\left(\sin^3\phi + \cos^3\phi\right) + \hat{\mathbf{z}}4, \\
P_3 &= \left(\sqrt{1^2+(-1)^2}, \tan^{-1}(-1/1), 2\right) = \left(\sqrt{2}, -45^\circ, 2\right), \\
\vec{C}(P_3) &= \hat{\mathbf{r}}0.707 + \hat{\mathbf{z}}4.
\end{aligned}$$

(d)

$$\begin{aligned}
\vec{D} &= (\hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta)\sin\theta + (\hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta)\cos\theta + \hat{\boldsymbol{\phi}}\cos^2\phi = \hat{\mathbf{r}} + \hat{\boldsymbol{\phi}}\cos^2\phi, \\
P_4 &= (2\sin(\pi/2), \pi/4, 2\cos(\pi/2)) = (2, 45^\circ, 0),
\end{aligned}$$

$$\vec{D}(P_4) = \hat{\mathbf{r}} + \hat{\boldsymbol{\phi}}\tfrac{1}{2}\,.$$

(e)

$$\begin{aligned}
\mathbf{E} &= (\hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta)\cos\phi + (\hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta)\sin\phi + \hat{\boldsymbol{\phi}}\sin^2\theta,\\
P_5 &= \left(3, \frac{\pi}{2}, \pi\right),\\
\mathbf{E}(P_5) &= \left(\hat{\mathbf{r}}\sin\frac{\pi}{2} + \hat{\mathbf{z}}\cos\frac{\pi}{2}\right)\cos\pi + \left(\hat{\mathbf{r}}\cos\frac{\pi}{2} - \hat{\mathbf{z}}\sin\frac{\pi}{2}\right)\sin\pi + \hat{\boldsymbol{\phi}}\sin^2\frac{\pi}{2} = -\hat{\mathbf{r}} + \hat{\boldsymbol{\phi}}.
\end{aligned}$$