3.38 The gradient of a scalar function T is given by

$$\nabla T = \hat{\mathbf{z}} e^{-4z}.$$

If $T = 10$ at $z = 0$, find $T(z)$.

Solution:

$$\nabla T = \hat{\mathbf{z}} e^{-4z}.$$

By choosing P_1 at $z = 0$ and P_2 at any point z, Eq. (3.76) becomes

$$\begin{aligned} T(z) - T(0) &= \int_0^z \nabla T \cdot d\mathbf{l}' = \int_0^z \hat{\mathbf{z}} e^{-4z'} \cdot (\hat{\mathbf{x}}\, dx' + \hat{\mathbf{y}}\, dy' + \hat{\mathbf{z}}\, dz') \\ &= \int_0^z e^{-4z'}\, dz' = -\left.\frac{-e^{-4z'}}{4}\right|_0^z = \frac{1}{4}(1 - e^{-4z}). \end{aligned}$$

Hence,

$$T(z) = T(0) + \frac{1}{4}(1 - e^{-4z}) = 10 + \frac{1}{4}(1 - e^{-4z}).$$