3.39 For the scalar function $V = xy^2 - z^2$, determine its directional derivative along the direction of vector $\mathbf{A} = (\hat{\mathbf{x}} - \hat{\mathbf{y}}z)$ and then evaluate it at $P = (1, -1, 2)$.

Solution: The directional derivative is given by Eq. (3.75) as $dV/dl = \nabla V \cdot \hat{\mathbf{a}}_l$, where the unit vector in the direction of $\vec{A}$ is given by Eq. (3.2):

$$\hat{\mathbf{a}}_l = \frac{\hat{\mathbf{x}} - \hat{\mathbf{y}}z}{\sqrt{1+z^2}},$$

and the gradient of V in Cartesian coordinates is given by Eq. (3.72):

$$\nabla V = \hat{\mathbf{x}}y^2 + \hat{\mathbf{y}}2xy - \hat{\mathbf{z}}2z.$$

Therefore, by Eq. (3.75),

$$\frac{dV}{dl} = \frac{y^2 - 2xyz}{\sqrt{1+z^2}}.$$

At $P = (1, -1, 2)$,

$$\left(\frac{dV}{dl}\right)\bigg|_{(1,-1,2)} = \frac{3}{\sqrt{5}} = 1.34.$$