3.4 Given $\mathbf{A} = \hat{\mathbf{x}}2 - \hat{\mathbf{y}}3 + \hat{\mathbf{z}}1$ and $\mathbf{B} = \hat{\mathbf{x}}B_x + \hat{\mathbf{y}}2 + \hat{\mathbf{z}}B_z$:

(a) find B_x and B_z if $\mathbf{A}$ is parallel to $\mathbf{B}$;

(b) find a relation between B_x and B_z if $\mathbf{A}$ is perpendicular to $\mathbf{B}$.

Solution:

(a) If $\vec{A}$ is parallel to $\vec{B}$, then their directions are equal or opposite: $\hat{\mathbf{a}}_A = \pm\hat{\mathbf{a}}_B$, or

$$\vec{A}/\left|\vec{A}\right| = \pm\vec{B}/\left|\vec{B}\right|,$$

$$\frac{\hat{\mathbf{x}}2 - \hat{\mathbf{y}}3 + \hat{\mathbf{z}}}{\sqrt{14}} = \pm\frac{\hat{\mathbf{x}}B_x + \hat{\mathbf{y}}2 + \hat{\mathbf{z}}B_z}{\sqrt{4 + B_x^2 + B_z^2}}.$$

From the y-component,

$$\frac{-3}{\sqrt{14}} = \frac{\pm 2}{\sqrt{4 + B_x^2 + B_z^2}}$$

which can only be solved for the minus sign (which means that $\vec{A}$ and $\vec{B}$ must point in opposite directions for them to be parallel). Solving for $B_x^2 + B_z^2$,

$$B_x^2 + B_z^2 = \left(\frac{-2}{-3}\sqrt{14}\right)^2 - 4 = \frac{20}{9}.$$

From the x-component,

$$\frac{2}{\sqrt{14}} = \frac{-B_x}{\sqrt{56/9}}, \qquad B_x = \frac{-2\sqrt{56}}{3\sqrt{14}} = \frac{-4}{3}$$

and, from the z-component,

$$B_z = \frac{-2}{3}.$$

This is consistent with our result for $B_x^2 + B_z^2$.

These results could also have been obtained by assuming θ_{AB} was $0°$ or $180°$ and solving $\left|\vec{A}\right|\left|\vec{B}\right| = \pm\vec{A}\cdot\vec{B}$, or by solving $\vec{A}\times\vec{B} = 0$.

(b) If $\vec{A}$ is perpendicular to $\vec{B}$, then their dot product is zero (see Section 3-1.4). Using Eq. (3.21),

$$0 = \vec{A}\cdot\vec{B} = 2B_x - 6 + B_z,$$

or

$$B_z = 6 - 2B_x.$$

There are an infinite number of vectors which could be $\vec{B}$ and be perpendicular to $\vec{A}$, but their x- and z-components must satisfy this relation.

This result could have also been obtained by assuming $\theta_{AB} = 90°$ and calculating $\left|\vec{A}\right|\left|\vec{B}\right| = \left|\vec{A}\times\vec{B}\right|$.