

3.42 For the scalar function $U = \frac{1}{R} \sin^2 \theta$, determine its directional derivative along the range direction $\hat{\mathbf{R}}$ and then evaluate it at $P = (5, \pi/4, \pi/2)$.

Solution:

$$U = \frac{1}{R} \sin^2 \theta,$$

$$\nabla U = \hat{\mathbf{R}} \frac{\partial U}{\partial R} + \hat{\boldsymbol{\theta}} \frac{1}{R} \frac{\partial U}{\partial \theta} + \hat{\boldsymbol{\phi}} \frac{1}{R \sin \theta} \frac{\partial U}{\partial \phi} = -\hat{\mathbf{R}} \frac{\sin^2 \theta}{R^2} - \hat{\boldsymbol{\theta}} \frac{2 \sin \theta \cos \theta}{R},$$

$$\frac{dU}{dl} = \nabla U \cdot \hat{\mathbf{R}} = -\frac{\sin^2 \theta}{R^2},$$

$$\left. \frac{dU}{dl} \right|_{(5, \pi/4, \pi/2)} = -\frac{\sin^2(\pi/4)}{25} = -0.02.$$
