

3.46 Vector field \mathbf{E} is characterized by the following properties: (a) \mathbf{E} points along $\hat{\mathbf{R}}$, (b) the magnitude of \mathbf{E} is a function of only the distance from the origin, (c) \mathbf{E} vanishes at the origin, and (d) $\nabla \cdot \mathbf{E} = 6$, everywhere. Find an expression for \mathbf{E} that satisfies these properties.

Solution: According to properties (a) and (b), \mathbf{E} must have the form

$$\mathbf{E} = \hat{\mathbf{R}}E_R$$

where E_R is a function of R only.

$$\nabla \cdot \mathbf{E} = \frac{1}{R^2} \frac{\partial}{\partial R} (R^2 E_R) = 6,$$

$$\frac{\partial}{\partial R} (R^2 E_R) = 6R^2,$$

$$\int_0^R \frac{\partial}{\partial R} (R^2 E_R) dR = \int_0^R 6R^2 dR,$$

$$R^2 E_R \Big|_0^R = \frac{6R^3}{3} \Big|_0^R,$$

$$R^2 E_R = 2R^3.$$

Hence,

$$E_R = 2R,$$

and

$$\mathbf{E} = \hat{\mathbf{R}}2R.$$
