

**3.51** Repeat Problem 3.50 for the contour shown in Fig. P3.50(b).

**Solution:** In addition to the independent condition that  $z = 0$ , the three lines of the triangle are represented by the equations  $y = 0$ ,  $y = 2 - x$ , and  $y = x$ , respectively.

(a)

$$\oint \vec{E} \cdot d\vec{l} = L_1 + L_2 + L_3,$$

$$\begin{aligned} L_1 &= \int (\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)) \cdot (\hat{\mathbf{x}}dx + \hat{\mathbf{y}}dy + \hat{\mathbf{z}}dz) \\ &= \int_{x=0}^2 (xy)|_{y=0, z=0} dx - \int_{y=0}^0 (x^2 + 2y^2)|_{z=0} dy + \int_{z=0}^0 (0)|_{y=0} dz = 0, \end{aligned}$$

$$\begin{aligned} L_2 &= \int (\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)) \cdot (\hat{\mathbf{x}}dx + \hat{\mathbf{y}}dy + \hat{\mathbf{z}}dz) \\ &= \int_{x=2}^1 (xy)|_{z=0, y=2-x} dx - \int_{y=0}^1 (x^2 + 2y^2)|_{x=2-y, z=0} dy + \int_{z=0}^0 (0)|_{y=2-x} dz \\ &= \left( x^2 - \frac{x^3}{3} \right) \Big|_{x=2}^1 - (4y - 2y^2 + y^3) \Big|_{y=0}^1 + 0 = \frac{-11}{3}, \end{aligned}$$

$$\begin{aligned} L_3 &= \int (\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)) \cdot (\hat{\mathbf{x}}dx + \hat{\mathbf{y}}dy + \hat{\mathbf{z}}dz) \\ &= \int_{x=1}^0 (xy)|_{y=x, z=0} dx - \int_{y=1}^0 (x^2 + 2y^2)|_{x=y, z=0} dy + \int_{z=0}^0 (0)|_{y=x} dz \\ &= \left( \frac{x^3}{3} \right) \Big|_{x=1}^0 - (y^3) \Big|_{y=1}^0 + 0 = \frac{2}{3}. \end{aligned}$$

Therefore,

$$\oint \vec{E} \cdot d\vec{l} = 0 - \frac{11}{3} + \frac{2}{3} = -3.$$

(b) From Eq. (3.105),  $\nabla \times \vec{E} = -\hat{\mathbf{z}}3x$ , so that

$$\begin{aligned} \iint \nabla \times \vec{E} \cdot d\vec{s} &= \int_{x=0}^1 \int_{y=0}^x ((-\hat{\mathbf{z}}3x) \cdot (\hat{\mathbf{z}}dy dx))|_{z=0} \\ &\quad + \int_{x=1}^2 \int_{y=0}^{2-x} ((-\hat{\mathbf{z}}3x) \cdot (\hat{\mathbf{z}}dy dx))|_{z=0} \\ &= - \int_{x=0}^1 \int_{y=0}^x 3x dy dx - \int_{x=1}^2 \int_{y=0}^{2-x} 3x dy dx \\ &= - \int_{x=0}^1 3x(x-0) dx - \int_{x=1}^2 3x((2-x)-0) dx \end{aligned}$$

$$= -(x^3)|_0^1 - (3x^2 - x^3)|_{x=1}^2 = -3.$$

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