

3.7 Given vectors $\mathbf{A} = \hat{x}2 - \hat{y} + \hat{z}3$ and $\mathbf{B} = \hat{x}3 - \hat{z}2$, find a vector \mathbf{C} whose magnitude is 9 and whose direction is perpendicular to both \mathbf{A} and \mathbf{B} .

Solution: The cross product of two vectors produces a new vector which is perpendicular to both of the original vectors. Two vectors exist which have a magnitude of 9 and are orthogonal to both \vec{A} and \vec{B} : one which is 9 units long in the direction of the unit vector parallel to $\vec{A} \times \vec{B}$, and one in the opposite direction.

$$\begin{aligned}\vec{C} &= \pm 9 \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} = \pm 9 \frac{(\hat{x}2 - \hat{y} + \hat{z}3) \times (\hat{x}3 - \hat{z}2)}{|(\hat{x}2 - \hat{y} + \hat{z}3) \times (\hat{x}3 - \hat{z}2)|} \\ &= \pm 9 \frac{\hat{x}2 + \hat{y}13 + \hat{z}3}{\sqrt{2^2 + 13^2 + 3^2}} \approx \pm (\hat{x}1.34 + \hat{y}8.67 + \hat{z}2.0) .\end{aligned}$$
