

4.10 A line of charge of uniform density ρ_ℓ occupies a semicircle of radius b as shown in Fig. P4.10. Use the material presented in Example 4-4 to determine the electric field at the origin.

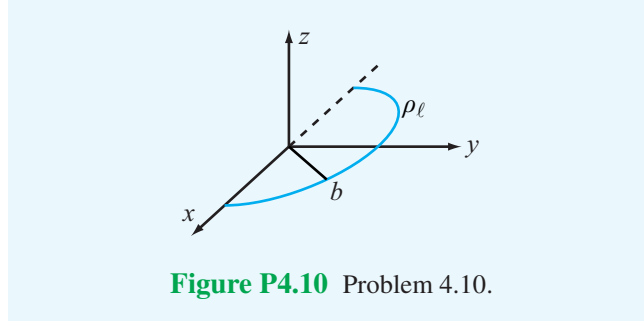


Figure P4.10 Problem 4.10.

Solution: Since we have only half of a circle, we need to integrate the expression for $d\mathbf{E}_1$ given in Example 4-4 over ϕ from 0 to π . Before we do that, however, we need to set $h = 0$ (the problem asks for \mathbf{E} at the origin). Hence,

$$\begin{aligned} d\mathbf{E}_1 &= \frac{\rho_\ell b}{4\pi\epsilon_0} \frac{(-\hat{\mathbf{r}}b + \hat{\mathbf{z}}h)}{(b^2 + h^2)^{3/2}} d\phi \Big|_{h=0} \\ &= \frac{-\hat{\mathbf{r}}\rho_\ell}{4\pi\epsilon_0 b} d\phi \\ \hat{\mathbf{r}} &= \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi. \end{aligned}$$

From symmetry $\hat{\mathbf{x}}$ components cancel.

$$\begin{aligned} d\mathbf{E}_1 &= \frac{-\hat{\mathbf{y}}\rho_\ell}{4\pi\epsilon_0 b} \sin\phi d\phi, \\ \mathbf{E}_1 &= \frac{-\hat{\mathbf{y}}\rho_\ell}{4\pi\epsilon_0 b} \int_0^\pi \sin\phi d\phi \\ &= \frac{\hat{\mathbf{y}}\rho_\ell}{4\pi\epsilon_0 b} \cos\phi \Big|_0^\pi \\ &= \frac{-\hat{\mathbf{y}}\rho_\ell}{2\pi\epsilon_0 b}. \end{aligned}$$
