

4.11 A square with sides of 2 m has a charge of $40 \mu\text{C}$ at each of its four corners. Determine the electric field at a point 5 m above the center of the square.

Solution: The distance $|R|$ between any of the charges and point P is

$$|R| = \sqrt{1^2 + 1^2 + 5^2} = \sqrt{27}.$$

$$\begin{aligned} \mathbf{E} &= \frac{Q}{4\pi\epsilon_0} \left[\frac{\mathbf{R}_1}{|\mathbf{R}|^3} + \frac{\mathbf{R}_2}{|\mathbf{R}|^3} + \frac{\mathbf{R}_3}{|\mathbf{R}|^3} + \frac{\mathbf{R}_4}{|\mathbf{R}|^3} \right] \\ &= \frac{Q}{4\pi\epsilon_0} \left[\frac{-\hat{\mathbf{x}} - \hat{\mathbf{y}} + \hat{\mathbf{z}}5}{(27)^{3/2}} + \frac{\hat{\mathbf{x}} - \hat{\mathbf{y}} + \hat{\mathbf{z}}5}{(27)^{3/2}} + \frac{-\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}5}{(27)^{3/2}} + \frac{\hat{\mathbf{x}} + \hat{\mathbf{y}} + \hat{\mathbf{z}}5}{(27)^{3/2}} \right] \\ &= \hat{\mathbf{z}} \frac{5Q}{(27)^{3/2}\pi\epsilon_0} = \hat{\mathbf{z}} \frac{5 \times 40 \mu\text{C}}{(27)^{3/2}\pi\epsilon_0} = \frac{1.42}{\pi\epsilon_0} \times 10^{-6} \text{ (V/m)} = \hat{\mathbf{z}} 51.2 \text{ (kV/m)}. \end{aligned}$$

