

4.14 A line of charge with uniform density $\rho_\ell = 8 \text{ } (\mu\text{C/m})$ exists in air along the z -axis between $z = 0$ and $z = 5 \text{ cm}$. Find \mathbf{E} at $(0, 10 \text{ cm}, 0)$.

Solution: Use of Eq. (4.21c) for the line of charge shown in Fig. P4.14 gives

$$\begin{aligned}\mathbf{E} &= \frac{1}{4\pi\epsilon_0} \int_{l'} \hat{\mathbf{R}}' \frac{\rho_\ell dl'}{R'^2}, \\ R' &= \hat{\mathbf{y}} 0.1 - \hat{\mathbf{z}} z \\ &= \frac{1}{4\pi\epsilon_0} \int_{z=0}^{0.05} (8 \times 10^{-6}) \frac{(\hat{\mathbf{y}} 0.1 - \hat{\mathbf{z}} z)}{[(0.1)^2 + z^2]^{3/2}} dz \\ &= \frac{8 \times 10^{-6}}{4\pi\epsilon_0} \left[\frac{\hat{\mathbf{y}} 10z + \hat{\mathbf{z}}}{\sqrt{(0.1)^2 + z^2}} \right] \bigg|_{z=0}^{0.05} \\ &= 71.86 \times 10^3 [\hat{\mathbf{y}} 4.47 - \hat{\mathbf{z}} 1.06] = \hat{\mathbf{y}} 321.4 \times 10^3 - \hat{\mathbf{z}} 76.2 \times 10^3 \quad (\text{V/m}).\end{aligned}$$

