4.19 Three infinite lines of charge, all parallel to the z-axis, are located at the three corners of the kite-shaped arrangement shown in Fig. P4.19. If the two right triangles are symmetrical and of equal corresponding sides, show that the electric field is zero at the origin.

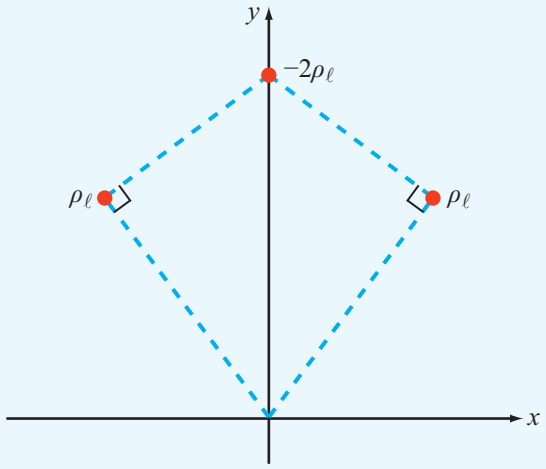

Figure P4.19 Kite-shaped arrangment of line charges for Problem 4.19.

Solution: The field due to an infinite line of charge is given by Eq. (4.33). In the present case, the total $\mathbf{E}$ at the origin is

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3.$$

The components of $\mathbf{E}_1$ and $\mathbf{E}_2$ along $\hat{\mathbf{x}}$ cancel and their components along $-\hat{\mathbf{y}}$ add. Also, $\mathbf{E}_3$ is along $\hat{\mathbf{y}}$ because the line charge on the y-axis is negative. Hence,

$$\mathbf{E} = -\hat{\mathbf{y}}\frac{2\rho_l \cos\theta}{2\pi\varepsilon_0 R_1} + \hat{\mathbf{y}}\frac{2\rho_l}{2\pi\varepsilon_0 R_2}.$$

But $\cos\theta = R_1/R_2$. Hence,

$$\mathbf{E} = -\hat{\mathbf{y}}\frac{\rho_l}{\pi\varepsilon_0 R_1}\frac{R_1}{R_2} + \hat{\mathbf{y}}\frac{\rho_l}{\pi\varepsilon_0 R_2} = 0.$$