

4.22 A horizontal strip lying in the x - y plane is of width d in the y -direction and infinitely long in the x -direction. If the strip is in air and has a uniform charge distribution ρ_s , use Coulomb's law to obtain an explicit expression for the electric field at a point P located at a distance h above the centerline of the strip. Extend your result to the special case where d is infinite and compare it with Eq. (4.25).

Solution:

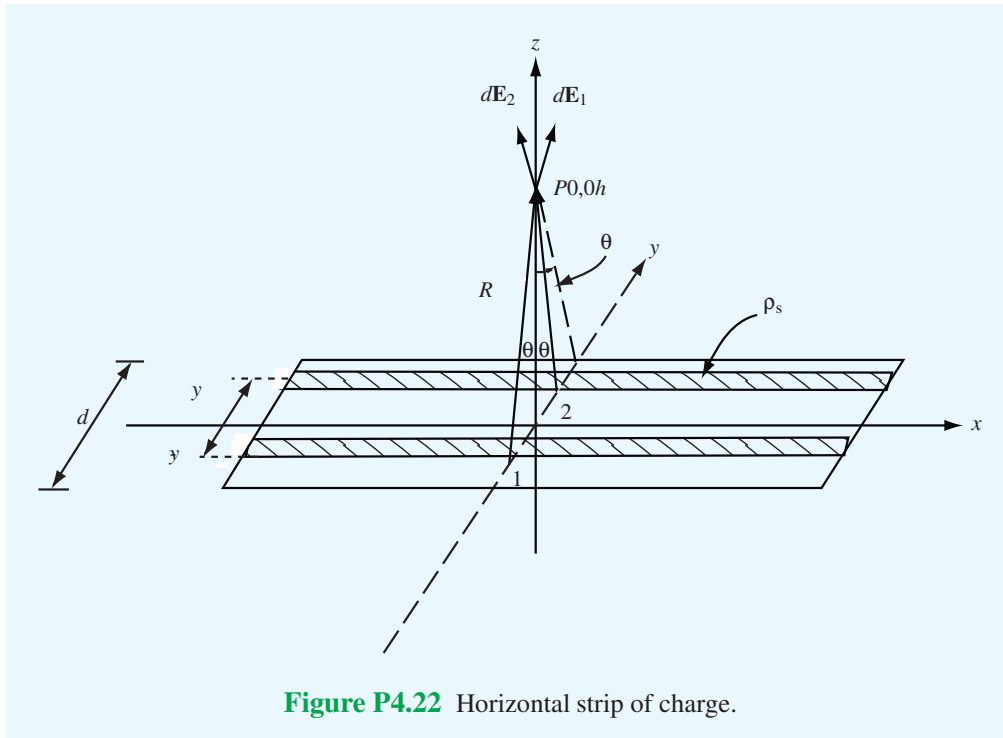


Figure P4.22 Horizontal strip of charge.

The strip of charge density ρ_s (C/m^2) can be treated as a set of adjacent line charges each of charge $\rho_l = \rho_s dy$ and width dy . At point P , the fields of line charge at distance y and line charge at distance $-y$ give contributions that cancel each other along \hat{y} and add along \hat{z} . For each such pair,

$$d\mathbf{E} = \hat{z} \frac{2\rho_s dy \cos \theta}{2\pi\epsilon_0 R}.$$

With $R = h/\cos \theta$, we integrate from $y = 0$ to $d/2$, which corresponds to $\theta = 0$ to $\theta_0 = \sin^{-1}[(d/2)/(h^2 + (d/2)^2)^{1/2}]$. Thus,

$$\mathbf{E} = \int_0^{d/2} d\mathbf{E} = \hat{z} \frac{\rho_s}{\pi\epsilon_0} \int_0^{d/2} \frac{\cos \theta}{R} dy = \hat{z} \frac{\rho_s}{\pi\epsilon_0} \int_0^{\theta_0} \frac{\cos^2 \theta}{h} \cdot \frac{h}{\cos^2 \theta} d\theta$$

$$= \hat{\mathbf{z}} \frac{\rho_s}{\pi \epsilon_0} \theta_0.$$

For an infinitely wide sheet, $\theta_0 = \pi/2$ and $\mathbf{E} = \hat{\mathbf{z}} \frac{\rho_s}{2\epsilon_0}$, which is identical with Eq. (4.25).
