4.24 Charge Q_1 is uniformly distributed over a thin spherical shell of radius a, and charge Q_2 is uniformly distributed over a second spherical shell of radius b, with $b > a$. Apply Gauss's law to find $\mathbf{E}$ in the regions $R < a$, $a < R < b$, and $R > b$.

Solution: Using symmetry considerations, we know $\mathbf{D} = \hat{\mathbf{R}} D_R$. From Table 3.1, $d\mathbf{s} = \hat{\mathbf{R}} R^2 \sin\theta \, d\theta \, d\phi$ for an element of a spherical surface. Using Gauss's law in integral form (Eq. (4.29)),

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q_{\text{tot}},$$

where Q_{tot} is the total charge enclosed in S. For a spherical surface of radius R,

$$\begin{aligned} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} (\hat{\mathbf{R}} D_R) \cdot (\hat{\mathbf{R}} R^2 \sin\theta \, d\theta \, d\phi) &= Q_{\text{tot}}, \\ D_R R^2 (2\pi) [-\cos\theta]_0^{\pi} &= Q_{\text{tot}}, \\ D_R &= \frac{Q_{\text{tot}}}{4\pi R^2} \,. \end{aligned}$$

From Eq. (4.15), we know a linear, isotropic material has the constitutive relationship $\mathbf{D} = \varepsilon \mathbf{E}$. Thus, we find $\mathbf{E}$ from $\mathbf{D}$.

(a) In the region $R < a$,

$$Q_{\text{tot}} = 0, \qquad \mathbf{E} = \hat{\mathbf{R}} E_R = \frac{\hat{\mathbf{R}} Q_{\text{tot}}}{4\pi R^2 \varepsilon} = 0 \qquad \text{(V/m)}.$$

(b) In the region $a < R < b$,

$$Q_{\text{tot}} = Q_1, \qquad \mathbf{E} = \hat{\mathbf{R}} E_R = \frac{\hat{\mathbf{R}} Q_1}{4\pi R^2 \varepsilon} \qquad \text{(V/m)}.$$

(c) In the region $R > b$,

$$Q_{\text{tot}} = Q_1 + Q_2, \qquad \mathbf{E} = \hat{\mathbf{R}} E_R = \frac{\hat{\mathbf{R}} (Q_1 + Q_2)}{4\pi R^2 \varepsilon} \qquad \text{(V/m)}.$$