

4.25 The electric flux density inside a dielectric sphere of radius a centered at the origin is given by

$$\mathbf{D} = \hat{\mathbf{R}}\rho_0 R \quad (\text{C/m}^2)$$

where ρ_0 is a constant. Find the total charge inside the sphere.

Solution:

$$\begin{aligned} Q &= \oint_S \mathbf{D} \cdot d\mathbf{s} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \hat{\mathbf{R}}\rho_0 R \cdot \hat{\mathbf{R}}R^2 \sin \theta \, d\theta \, d\phi \Big|_{R=a} \\ &= 2\pi\rho_0 a^3 \int_0^{\pi} \sin \theta \, d\theta = -2\pi\rho_0 a^3 \cos \theta \Big|_0^{\pi} = 4\pi\rho_0 a^3 \quad (\text{C}). \end{aligned}$$
