4.26 An infinitely long cylindrical shell extending between $r = 1$ m and $r = 2$ m contains a uniform charge density ρ_{v0}. Apply Gauss's law to find $\mathbf{D}$ in all regions.

Solution: For $r < 1$ m, $\mathbf{D} = 0$.

For $1 \le r \le 2$ m,

$$\oint_S \hat{\mathbf{r}} D_r \cdot d\mathbf{s} = Q,$$

$$D_r \cdot 2\pi rL = \rho_{v0} \cdot \pi L(r^2 - 1^2),$$

$$\mathbf{D} = \hat{\mathbf{r}} D_r = \hat{\mathbf{r}} \frac{\rho_{v0}\pi L(r^2-1)}{2\pi rL} = \hat{\mathbf{r}} \frac{\rho_{v0}(r^2-1)}{2r}, \qquad 1 \le r \le 2 \text{ m}.$$

For $r \ge 2$ m,

$$D_r \cdot 2\pi rL = \rho_{v0}\pi L(2^2 - 1^2) = 3\rho_{v0}\pi L,$$

$$\mathbf{D} = \hat{\mathbf{r}} D_r = \hat{\mathbf{r}} \frac{3\rho_{v0}}{2r}, \qquad r \ge 2 \text{ m}.$$

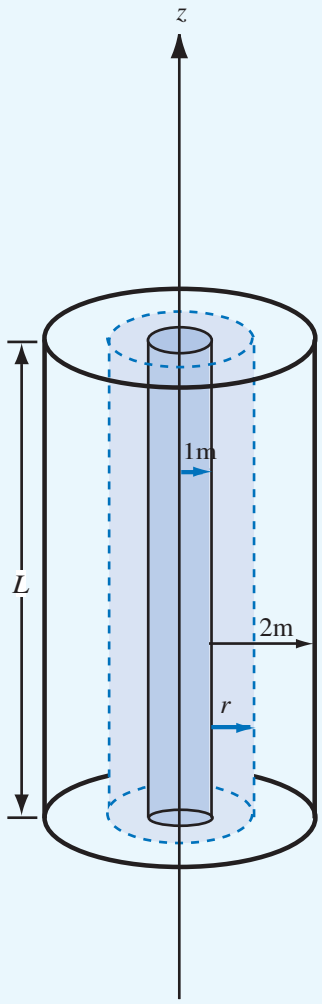

Figure P4.26 Cylindrical shell.