

4.27 In a certain region of space, the charge density is given in cylindrical coordinates by the function:

$$\rho_v = 50re^{-r} \quad (\text{C/m}^3)$$

Apply Gauss's law to find \mathbf{D} .

Solution:

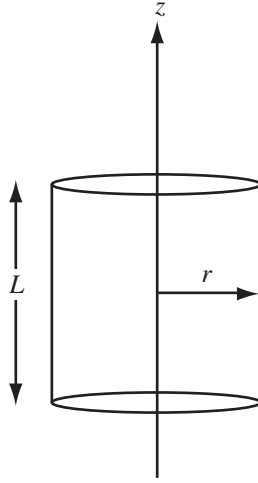


Figure P4.27 Gaussian surface.

Method 1: Integral Form of Gauss's Law

Since ρ_v varies as a function of r only, so will \mathbf{D} . Hence, we construct a cylinder of radius r and length L , coincident with the z -axis. Symmetry suggests that \mathbf{D} has the functional form $\mathbf{D} = \hat{\mathbf{r}}D$. Hence,

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q,$$

$$\int \hat{\mathbf{r}}D \cdot d\mathbf{s} = D(2\pi rL),$$

$$Q = 2\pi L \int_0^r 50re^{-r} \cdot r \, dr$$

$$= 100\pi L[-r^2e^{-r} + 2(1 - e^{-r}(1 + r))],$$

$$\mathbf{D} = \hat{\mathbf{r}}D = \hat{\mathbf{r}}50 \left[\frac{2}{r}(1 - e^{-r}(1 + r)) - re^{-r} \right].$$

Method 2: Differential Method

$$\nabla \cdot \mathbf{D} = \rho_v, \quad \mathbf{D} = \hat{\mathbf{r}} D_r,$$

with D_r being a function of r .

$$\frac{1}{r} \frac{\partial}{\partial r} (r D_r) = 50 r e^{-r},$$

$$\frac{\partial}{\partial r} (r D_r) = 50 r^2 e^{-r},$$

$$\int_0^r \frac{\partial}{\partial r} (r D_r) dr = \int_0^r 50 r^2 e^{-r} dr,$$

$$r D_r = 50 [2(1 - e^{-r}(1 + r)) - r^2 e^{-r}],$$

$$\mathbf{D} = \hat{\mathbf{r}} r D_r = \hat{\mathbf{r}} 50 \left[\frac{2}{r} (1 - e^{-r}(1 + r)) - r e^{-r} \right].$$
