4.32 Show that the electric potential difference V_{12} between two points in air at radial distances r_1 and r_2 from an infinite line of charge with density ρ_ℓ along the z axis is $V_{12} = (\rho_\ell/2\pi\varepsilon_0)\ln(r_2/r_1)$.

Solution: From Eq. (4.33), the electric field due to an infinite line of charge is

$$\mathbf{E} = \hat{\mathbf{r}}E_{\mathrm{r}} = \hat{\mathbf{r}}\,\frac{\rho_l}{2\pi\varepsilon_0 r}\,.$$

Hence, the potential difference is

$$V_{12} = -\int_{r_2}^{r_1} \mathbf{E}\cdot d\mathbf{l} = -\int_{r_2}^{r_1} \frac{\hat{\mathbf{r}}\rho_l}{2\pi\varepsilon_0 r}\cdot\hat{\mathbf{r}}\,dr = \frac{\rho_l}{2\pi\varepsilon_0}\ln\left(\frac{r_2}{r_1}\right).$$