4.37 Two infinite lines of charge, both parallel to the z-axis, lie in the x–z plane, one with density ρ_ℓ and located at $x = a$ and the other with density $-\rho_\ell$ and located at $x = -a$. Obtain an expression for the electric potential $V(x, y)$ at a point $P = (x, y)$ relative to the potential at the origin.

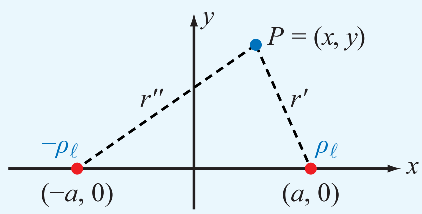

Figure P4.37 Problem 4.37.

Solution: According to the result of Problem 4.33, the electric potential difference between a point at a distance r_1 and another at a distance r_2 from a line charge of density ρ_l is

$$V = \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{r_2}{r_1}\right).$$

Applying this result to the line charge at $x = a$, which is at a distance a from the origin:

$$\begin{aligned} V' &= \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{r'}\right) \qquad (r_2 = a \text{ and } r_1 = r') \\ &= \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{\sqrt{(x-a)^2 + y^2}}\right). \end{aligned}$$

Similarly, for the negative line charge at $x = -a$,

$$\begin{aligned} V'' &= \frac{-\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{r''}\right) \qquad (r_2 = a \text{ and } r_1 = r') \\ &= \frac{-\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{\sqrt{(x+a)^2 + y^2}}\right). \end{aligned}$$

The potential due to both lines is

$$V = V' + V'' = \frac{\rho_l}{2\pi\varepsilon_0} \left[\ln\left(\frac{a}{\sqrt{(x-a)^2 + y^2}}\right) - \ln\left(\frac{a}{\sqrt{(x+a)^2 + y^2}}\right) \right].$$

At the origin, $V = 0$, as it should be since the origin is the reference point. The potential is also zero along all points on the y-axis ($x = 0$).