4.39 The x–y plane contains a uniform sheet of charge with $\rho_{s_1} = 0.2$ (nC/m^2). A second sheet with $\rho_{s_2} = -0.2$ (nC/m^2) occupies the plane $z = 6$ m. Find V_{AB}, V_{BC}, and V_{AC} for $A(0,0,6\text{ m})$, $B(0,0,0)$, and $C(0,-2\text{ m},2\text{ m})$.

Solution: We start by finding the **E** field in the region between the plates. For any point above the x–y plane, $\mathbf{E}_1$ due to the charge on x–y plane is, from Eq. (4.25),

$$\mathbf{E}_1 = \hat{\mathbf{z}}\,\frac{\rho_{s_1}}{2\varepsilon_0}\,.$$

In the region below the top plate, **E** would point downwards for positive ρ_{s_2} on the top plate. In this case, $\rho_{s_2} = -\rho_{s_1}$. Hence,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = \hat{\mathbf{z}}\,\frac{\rho_{s_1}}{2\varepsilon_0} - \hat{\mathbf{z}}\,\frac{\rho_{s_2}}{2\varepsilon_0} = \hat{\mathbf{z}}\,\frac{2\rho_{s_1}}{2\varepsilon_0} = \hat{\mathbf{z}}\,\frac{\rho_{s_1}}{\varepsilon_0}\,.$$

Since **E** is along $\hat{\mathbf{z}}$, only change in position along z can result in change in voltage.

$$V_{AB} = -\int_0^6 \hat{\mathbf{z}}\,\frac{\rho_{s_1}}{\varepsilon_0}\cdot\hat{\mathbf{z}}\,dz = -\frac{\rho_{s_1}}{\varepsilon_0}\,z\bigg|_0^6 = -\frac{6\rho_{s_1}}{\varepsilon_0} = -\frac{6\times 0.2\times 10^{-9}}{8.85\times 10^{-12}} = -135.59\text{ V}.$$

The voltage at C depends only on the z-coordinate of C. Hence, with point A being at the lowest potential and B at the highest potential,

$$V_{BC} = \frac{-2}{6}\,V_{AB} = -\frac{(-135.59)}{3} = 45.20\text{ V},$$
$$V_{AC} = V_{AB} + V_{BC} = -135.59 + 45.20 = -90.39\text{ V}.$$

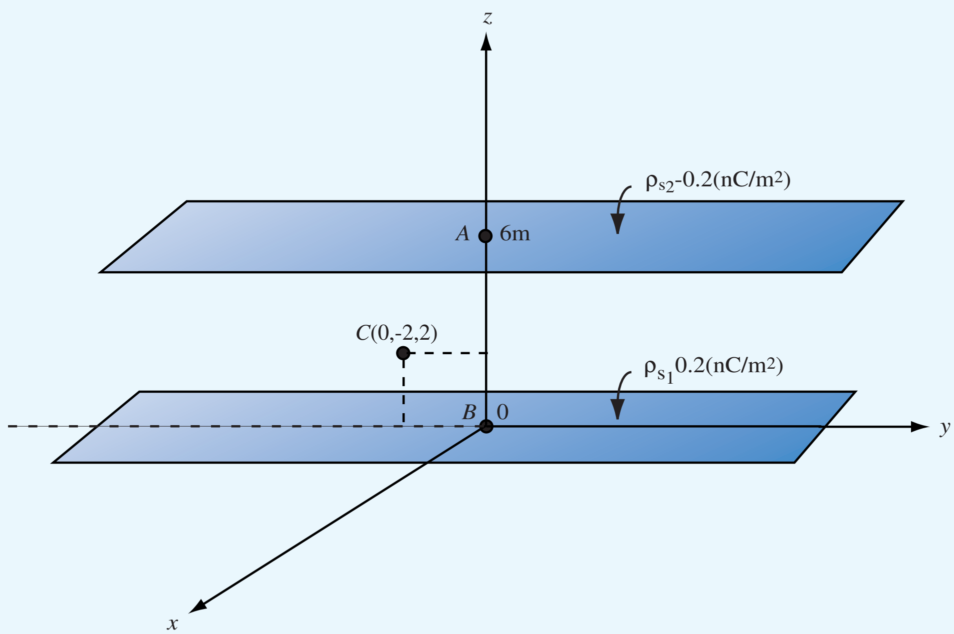

Figure P4.39 Two parallel planes of charge.