4.56 In a dielectric medium with $\varepsilon_r = 4$, the electric field is given by

$$\mathbf{E} = \hat{\mathbf{x}}(x^2 + 2z) + \hat{\mathbf{y}}x^2 - \hat{\mathbf{z}}(y+z) \qquad \text{(V/m)}$$

Calculate the electrostatic energy stored in the region $-1\text{ m} \le x \le 1\text{ m}$, $0 \le y \le 2\text{ m}$, and $0 \le z \le 3\text{ m}$.

Solution: Electrostatic potential energy is given by Eq. (4.124),

$$W_e = \frac{1}{2}\int_{\mathscr{V}} \varepsilon |\mathbf{E}|^2 \, d\mathscr{V} = \frac{\varepsilon}{2}\int_{z=0}^{3}\int_{y=0}^{2}\int_{x=-1}^{1} [(x^2+2z)^2 + x^4 + (y+z)^2]\, dx\, dy\, dz$$

$$= \frac{4\varepsilon_0}{2}\left(\left(\left(\frac{2}{5}x^5yz + \frac{2}{3}z^2x^3y + \frac{4}{3}z^3xy + \frac{1}{12}(y+z)^4x\right)\Big|_{x=-1}^{1}\right)\Big|_{y=0}^{2}\right)\Big|_{z=0}^{3}$$

$$= \frac{4\varepsilon_0}{2}\left(\frac{1304}{5}\right) = 4.62 \times 10^{-9} \qquad \text{(J)}.$$