4.8 A circular beam of charge of radius a consists of electrons moving with a constant speed u along the $+z$ direction. The beam's axis is coincident with the z axis and the electron charge density is given by

$$\rho_v = -cr^2 \qquad \text{(c/m}^3\text{)}$$

where c is a constant and r is the radial distance from the axis of the beam.

(a) Determine the charge density per unit length.

(b) Determine the current crossing the z plane.

Solution:

(a)

$$\begin{aligned}
\rho_l &= \int \rho_v \, ds \\
&= \int_{r=0}^{a} \int_{\phi=0}^{2\pi} -cr^2 \cdot r \, dr \, d\phi = -2\pi c \left. \frac{r^4}{4} \right|_0^a = -\frac{\pi c a^4}{2} \qquad \text{(C/m)}.
\end{aligned}$$

(b)

$$\begin{aligned}
\mathbf{J} &= \rho_v \mathbf{u} = -\hat{\mathbf{z}} c r^2 u \qquad \text{(A/m}^2\text{)} \\
I &= \int \mathbf{J} \cdot d\mathbf{s} \\
&= \int_{r=0}^{a} \int_{\phi=0}^{2\pi} (-\hat{\mathbf{z}} c u r^2) \cdot \hat{\mathbf{z}} \, r \, dr \, d\phi \\
&= -2\pi c u \int_0^a r^3 \, dr = -\frac{\pi c u a^4}{2} = \rho_l u. \qquad \text{(A)}.
\end{aligned}$$