5.15 A circular loop of radius a carrying current I_1 is located in the x–y plane as shown in Fig. P5.15. In addition, an infinitely long wire carrying current I_2 in a direction parallel with the z-axis is located at $y = y_0$.

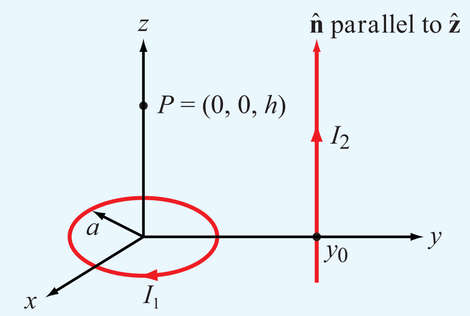

Figure P5.15 Problem 5.15.

(a) Determine **H** at $P = (0,0,h)$.

(b) Evaluate **H** for $a = 3$ cm, $y_0 = 10$ cm, $h = 4$ cm, $I_1 = 10$ A, and $I_2 = 20$ A.

Solution:

(a) The magnetic field at $P = (0,0,h)$ is composed of $\mathbf{H}_1$ due to the loop and $\mathbf{H}_2$ due to the wire:

$$\mathbf{H} = \mathbf{H}_1 + \mathbf{H}_2.$$

From (5.34), with $z = h$,

$$\mathbf{H}_1 = \hat{\mathbf{z}} \frac{I_1 a^2}{2(a^2+h^2)^{3/2}} \qquad \text{(A/m)}.$$

From (5.30), the field due to the wire at a distance $r = y_0$ is

$$\mathbf{H}_2 = \hat{\boldsymbol{\phi}} \frac{I_2}{2\pi y_0}$$

where $\hat{\boldsymbol{\phi}}$ is defined with respect to the coordinate system of the wire. Point P is located at an angel $\phi = -90^\circ$ with respect to the wire coordinates. From Table 3-2,

$$\begin{aligned} \hat{\boldsymbol{\phi}} &= -\hat{\mathbf{x}} \sin\phi + \hat{\mathbf{y}} \cos\phi \\ &= \hat{\mathbf{x}} \qquad (\text{at } \phi = -90^\circ). \end{aligned}$$

Hence,

$$\mathbf{H} = \hat{\mathbf{z}} \frac{I_1 a^2}{2(a^2+h^2)^{3/2}} + \hat{\mathbf{x}} \frac{I_2}{2\pi y_0} \qquad \text{(A/m)}.$$

(b)

$$\mathbf{H} = \hat{\mathbf{z}}36 + \hat{\mathbf{x}}31.83 \qquad \text{(A/m)}.$$