5.16 The long, straight conductor shown in Fig. P5.16 lies in the plane of the rectangular loop at a distance $d = 0.1$ m. The loop has dimensions $a = 0.2$ m and $b = 0.5$ m, and the currents are $I_1 = 40$ A and $I_2 = 30$ A. Determine the net magnetic force acting on the loop.

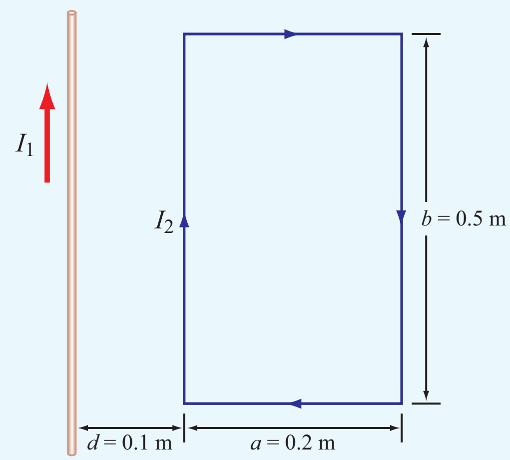

Figure P5.16 Current loop next to a conducting wire (Problem 5.16).

Solution: The net magnetic force on the loop is due to the magnetic field surrounding the wire carrying current I_1. The magnetic forces on the loop as a whole due to the current in the loop itself are canceled out by symmetry. Consider the wire carrying I_1 to coincide with the z-axis, and the loop to lie in the $+x$ side of the x-z plane. Assuming the wire and the loop are surrounded by free space or other nonmagnetic material, Eq. (5.30) gives

$$\vec{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r}.$$

In the plane of the loop, this magnetic field is

$$\vec{B} = \hat{\mathbf{y}} \frac{\mu_0 I_1}{2\pi x}.$$

Then, from Eq. (5.12), the force on the side of the loop nearest the wire is

$$\vec{F}_{\text{m1}} = I_2 \boldsymbol{\ell} \times \vec{B} = I_2 (\hat{\mathbf{z}} b) \times \left(\hat{\mathbf{y}} \frac{\mu_0 I_1}{2\pi x} \right) \bigg|_{x=d} = -\hat{\mathbf{x}} \frac{\mu_0 I_1 I_2 b}{2\pi d}.$$

The force on the side of the loop farthest from the wire is

$$\vec{F}_{\mathrm{m2}} = I_2 \boldsymbol{\ell} \times \vec{B} = I_2 \left(-\hat{\mathbf{z}} b\right) \times \left. \left(\hat{\mathbf{y}} \frac{\mu_0 I_1}{2\pi x}\right) \right|_{x=a+d} = \hat{\mathbf{x}} \frac{\mu_0 I_1 I_2 b}{2\pi \left(a+d\right)}.$$

The other two sides do not contribute any net forces to the loop because they are equal in magnitude and opposite in direction. Therefore, the total force on the loop is

$$\begin{aligned}
\vec{F} &= \vec{F}_{\mathrm{m1}} + \vec{F}_{\mathrm{m2}} \\
&= -\hat{\mathbf{x}} \frac{\mu_0 I_1 I_2 b}{2\pi d} + \hat{\mathbf{x}} \frac{\mu_0 I_1 I_2 b}{2\pi \left(a+d\right)} \\
&= -\hat{\mathbf{x}} \frac{\mu_0 I_1 I_2 a b}{2\pi d \left(a+d\right)} \\
&= -\hat{\mathbf{x}} \frac{4\pi \times 10^{-7} \times 40 \times 30 \times 0.2 \times 0.5}{2\pi \times 0.1 \times 0.3} = -\hat{\mathbf{x}} 0.8 \qquad \text{(mN)}.
\end{aligned}$$

The force is pulling the loop toward the wire.