5.17 In the arrangement shown in Fig. P5.17, each of the two long, parallel conductors carries a current I, is supported by 8-cm-long strings, and has a mass per unit length of 1.2 g/cm. Due to the repulsive force acting on the conductors, the angle θ between the supporting strings is 10°. Determine the magnitude of I and the relative directions of the currents in the two conductors.

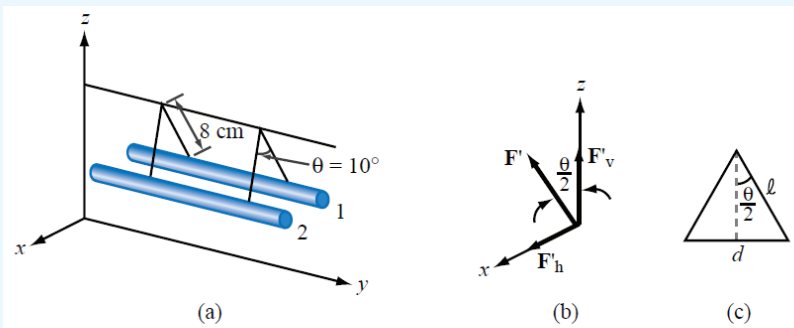

Figure P5.17 Parallel conductors supported by strings (Problem 5.17).

Solution: While the vertical component of the tension in the strings is counteracting the force of gravity on the wires, the horizontal component of the tension in the strings is counteracting the magnetic force, which is pushing the wires apart. According to Section 5-3, the magnetic force is repulsive when the currents are in opposite directions.

Figure P5.16(b) shows forces on wire 1 of part (a). The quantity $\mathbf{F}'$ is the tension force per unit length of wire due to the mass per unit length $m' = 1.2$ g/cm $= 0.12$ kg/m. The vertical component of $\mathbf{F}'$ balances out the gravitational force,

$$F'_{\text{v}} = m'g, \tag{23}$$

where $g = 9.8$ (m/s^2). But

$$F'_{\text{v}} = F' \cos(\theta/2). \tag{24}$$

Hence,

$$F' = \frac{m'g}{\cos(\theta/2)} \,. \tag{25}$$

The horizontal component of $\mathbf{F}'$ must be equal to the repulsion magnitude force given by Eq. (5.42):

$$F'_{\text{h}} = \frac{\mu_0 I^2}{2\pi d} = \frac{\mu_0 I^2}{2\pi[2\ell \sin(\theta/2)]} \,, \tag{26}$$

where d is the spacing between the wires and ℓ is the length of the string, as shown in Fig. P5.16(c). From Fig. 5.16(b),

$$F'_{\mathrm{h}} = F' \sin(\theta/2) = \frac{m'g}{\cos(\theta/2)} \sin(\theta/2) = m'g \tan(\theta/2). \tag{27}$$

Equating Eqs. (26) and (27) and then solving for I, we have

$$I = \sin(\theta/2) \sqrt{\frac{4\pi\ell m'g}{\mu_0 \cos(\theta/2)}} = \sin 5^\circ \sqrt{\frac{4\pi \times 0.08 \times 0.12 \times 9.8}{4\pi \times 10^{-7} \cos 5^\circ}} = 84.8 \qquad \text{(A)}.$$