5.22 Repeat Problem 5.21 for a current density $\mathbf{J} = \hat{\mathbf{z}} J_0 e^{-r}$.

Solution:

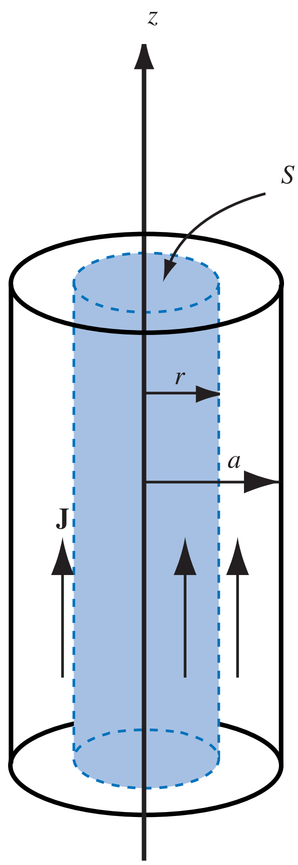

Figure P5.22 Cylindrical current.

(a) For $r \le a$, Ampère's law is

$$\oint_c \mathbf{H} \cdot d\mathbf{l} = I = \int_S \mathbf{J} \cdot d\mathbf{s},$$

$$\hat{\boldsymbol{\phi}} H \cdot \hat{\boldsymbol{\phi}}\, 2\pi r = \int_0^r \mathbf{J} \cdot d\mathbf{s} = \int_0^r \hat{\mathbf{z}} J_0 e^{-r} \cdot \hat{\mathbf{z}}\, 2\pi r\, dr,$$

$$2\pi r H = 2\pi J_0 \int_0^r r e^{-r}\, dr$$

$$= 2\pi J_0 [-e^{-r}(r+1)]_0^r = 2\pi J_0 [1 - e^{-r}(r+1)].$$

Hence,

$$\mathbf{H} = \hat{\boldsymbol{\phi}} H = \hat{\boldsymbol{\phi}} \frac{J_0}{r} [1 - e^{-r}(r+1)], \qquad \text{for } r \leq a.$$

(b) For $r \geq a$,

$$2\pi r H = 2\pi J_0 [-e^{-r}(r+1)]_0^a = 2\pi J_0 [1 - e^{-a}(a+1)],$$

$$\mathbf{H} = \hat{\boldsymbol{\phi}} H = \hat{\boldsymbol{\phi}} \frac{J_0}{r} \left[1 - e^{-a}(a+1)\right], \qquad r \geq a.$$