5.23 Current I flows along the positive z-direction in the inner conductor of a long coaxial cable and returns through the outer conductor. The inner conductor has radius a, and the inner and outer radii of the outer conductor are b and c, respectively.

(a) Determine the magnetic field in each of the following regions: $0 \leq r \leq a$, $a \leq r \leq b$, $b \leq r \leq c$, and $r \geq c$.

(b) Plot the magnitude of $\mathbf{H}$ as a function of r over the range from $r = 0$ to $r = 10$ cm, given that $I = 10$ A, $a = 2$ cm, $b = 4$ cm, and $c = 5$ cm.

Solution:

(a) Following the solution to Example 5-5, the magnetic field in the region $r < a$,

$$\vec{H} = \hat{\boldsymbol{\phi}} \frac{rI}{2\pi a^2},$$

and in the region $a < r < b$,

$$\vec{H} = \hat{\boldsymbol{\phi}} \frac{I}{2\pi r}.$$

The total area of the outer conductor is $A = \pi\left(c^2 - b^2\right)$ and the fraction of the area of the outer conductor enclosed by a circular contour centered at $r = 0$ in the region $b < r < c$ is

$$\frac{\pi\left(r^2 - b^2\right)}{\pi\left(c^2 - b^2\right)} = \frac{r^2 - b^2}{c^2 - b^2}.$$

The total current enclosed by a contour of radius r is therefore

$$I_{\text{enclosed}} = I\left(1 - \frac{r^2 - b^2}{c^2 - b^2}\right) = I\frac{c^2 - r^2}{c^2 - b^2},$$

and the resulting magnetic field is

$$\vec{H} = \hat{\boldsymbol{\phi}} \frac{I_{\text{enclosed}}}{2\pi r} = \hat{\boldsymbol{\phi}} \frac{I}{2\pi r}\left(\frac{c^2 - r^2}{c^2 - b^2}\right).$$

For $r > c$, the total enclosed current is zero: the total current flowing on the inner conductor is equal to the total current flowing on the outer conductor, but they are flowing in opposite directions. Therefore, $\vec{H} = 0$.

(b) See Fig. P5.23.

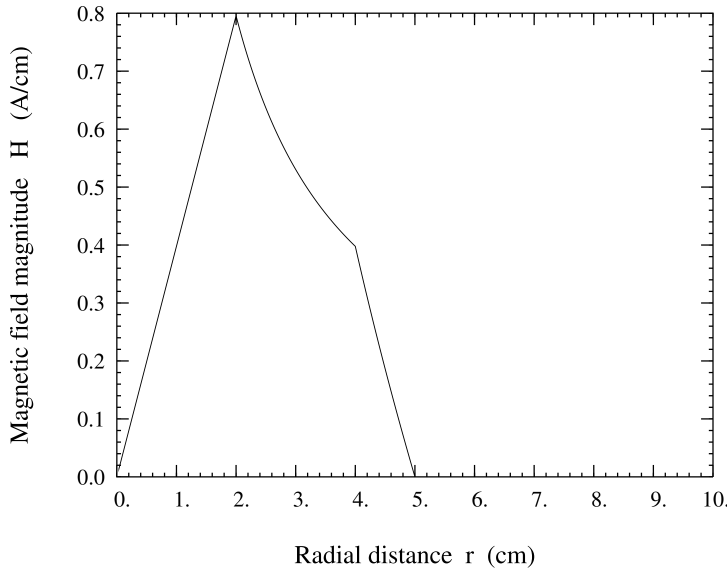

Figure P5.23: Problem 5.23.