

5.28 In a given region of space, the vector magnetic potential is given by $\mathbf{A} = \hat{\mathbf{x}}5 \cos \pi y + \hat{\mathbf{z}}(2 + \sin \pi x)$ (Wb/m).

- (a) Determine \mathbf{B} .
- (b) Use Eq. (5.66) to calculate the magnetic flux passing through a square loop with 0.25-m-long edges if the loop is in the x - y plane, its center is at the origin, and its edges are parallel to the x - and y -axes.
- (c) Calculate Φ again using Eq. (5.67).

Solution:

(a) From Eq. (5.53), $\vec{B} = \nabla \times \vec{A} = \hat{\mathbf{z}}5\pi \sin \pi y - \hat{\mathbf{y}}\pi \cos \pi x$.

(b) From Eq. (5.66),

$$\begin{aligned}\Phi &= \iint \vec{B} \cdot d\vec{S} = \int_{y=-0.125}^{0.125} \int_{x=-0.125}^{0.125} (\hat{\mathbf{z}}5\pi \sin \pi y - \hat{\mathbf{y}}\pi \cos \pi x) \cdot (\hat{\mathbf{z}} dx dy) \\ &= \left(\left(-5\pi x \frac{\cos \pi y}{\pi} \right) \Big|_{x=-0.125}^{0.125} \right) \Big|_{y=-0.125}^{0.125} \\ &= \frac{-5}{4} \left(\cos \left(\frac{\pi}{8} \right) - \cos \left(\frac{-\pi}{8} \right) \right) = 0.\end{aligned}$$

(c) From Eq. (5.67), $\Phi = \oint_C \mathbf{A} \cdot d\boldsymbol{\ell}$, where C is the square loop in the x - y plane with sides of length 0.25 m centered at the origin. Thus, the integral can be written as

$$\Phi = \oint_C \mathbf{A} \cdot d\boldsymbol{\ell} = S_{\text{front}} + S_{\text{back}} + S_{\text{left}} + S_{\text{right}},$$

where S_{front} , S_{back} , S_{left} , and S_{right} are the sides of the loop.

$$\begin{aligned}S_{\text{front}} &= \int_{x=-0.125}^{0.125} (\hat{\mathbf{x}}5 \cos \pi y + \hat{\mathbf{z}}(2 + \sin \pi x)) \Big|_{y=-0.125} \cdot (\hat{\mathbf{x}} dx) \\ &= \int_{x=-0.125}^{0.125} 5 \cos \pi y \Big|_{y=-0.125} dx \\ &= \left((5x \cos \pi y) \Big|_{y=-0.125} \right) \Big|_{x=-0.125}^{0.125} = \frac{5}{4} \cos \left(\frac{-\pi}{8} \right) = \frac{5}{4} \cos \left(\frac{\pi}{8} \right), \\ S_{\text{back}} &= \int_{x=-0.125}^{0.125} (\hat{\mathbf{x}}5 \cos \pi y + \hat{\mathbf{z}}(2 + \sin \pi x)) \Big|_{y=0.125} \cdot (-\hat{\mathbf{x}} dx) \\ &= - \int_{x=-0.125}^{0.125} 5 \cos \pi y \Big|_{y=0.125} dx \\ &= \left((-5x \cos \pi y) \Big|_{y=0.125} \right) \Big|_{x=-0.125}^{0.125} = -\frac{5}{4} \cos \left(\frac{\pi}{8} \right),\end{aligned}$$

$$\begin{aligned}
S_{\text{left}} &= \int_{y=-0.125}^{0.125} (\hat{\mathbf{x}}5 \cos \pi y + \hat{\mathbf{z}}(2 + \sin \pi x))|_{x=-0.125} \cdot (-\hat{\mathbf{y}} dy) \\
&= - \int_{y=-0.125}^{0.125} 0|_{x=-0.125} dy = 0, \\
S_{\text{right}} &= \int_{y=-0.125}^{0.125} (\hat{\mathbf{x}}5 \cos \pi y + \hat{\mathbf{z}}(2 + \sin \pi x))|_{x=0.125} \cdot (\hat{\mathbf{y}} dy) \\
&= \int_{y=-0.125}^{0.125} 0|_{x=0.125} dy = 0.
\end{aligned}$$

Thus,

$$\Phi = \oint_c \mathbf{A} \cdot d\boldsymbol{\ell} = S_{\text{front}} + S_{\text{back}} + S_{\text{left}} + S_{\text{right}} = \frac{5}{4} \cos\left(\frac{\pi}{8}\right) - \frac{5}{4} \cos\left(\frac{\pi}{8}\right) + 0 + 0 = 0.$$
