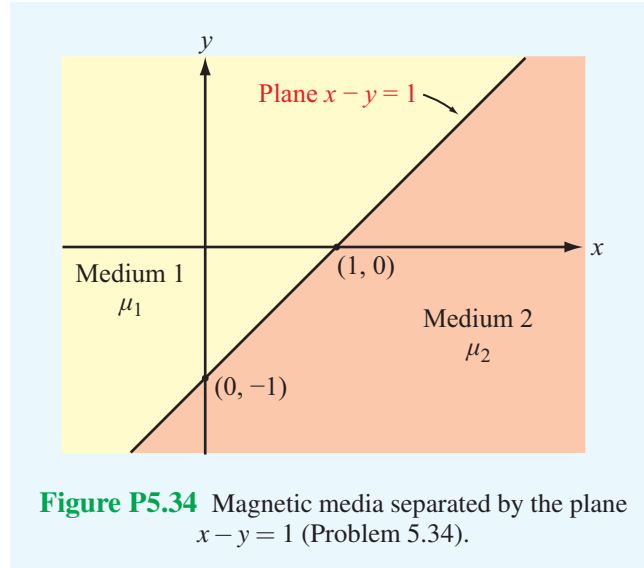


**5.34** In Fig. P5.34, the plane defined by  $x - y = 1$  separates medium 1 of permeability  $\mu_1$  from medium 2 of permeability  $\mu_2$ . If no surface current exists on the boundary and

$$\mathbf{B}_1 = \hat{\mathbf{x}}2 + \hat{\mathbf{y}}3 \quad (\text{T})$$

find  $\mathbf{B}_2$  and then evaluate your result for  $\mu_1 = 5\mu_2$ .

Hint: Start by deriving the equation for the unit vector normal to the given plane.



**Solution:** We need to find  $\hat{\mathbf{n}}_2$ . To do so, we start by finding any two vectors in the plane  $x - y = 1$ , and to do that, we need three non-collinear points in that plane. We choose  $(0, -1, 0)$ ,  $(1, 0, 0)$ , and  $(1, 0, 1)$ .

Vector  $\mathbf{A}_1$  is from  $(0, -1, 0)$  to  $(1, 0, 0)$ :

$$\mathbf{A}_1 = \hat{\mathbf{x}}1 + \hat{\mathbf{y}}1.$$

Vector  $\mathbf{A}_2$  is from  $(1, 0, 0)$  to  $(1, 0, 1)$ :

$$\mathbf{A}_2 = \hat{\mathbf{z}}1.$$

Hence, if we take the cross product  $\mathbf{A}_2 \times \mathbf{A}_1$ , we end up in a direction normal to the given plane, from medium 2 to medium 1,

$$\hat{\mathbf{n}}_2 = \frac{\mathbf{A}_2 \times \mathbf{A}_1}{|\mathbf{A}_2 \times \mathbf{A}_1|} = \frac{\hat{\mathbf{z}}1 \times (\hat{\mathbf{x}}1 + \hat{\mathbf{y}}1)}{|\mathbf{A}_2 \times \mathbf{A}_1|} = \frac{\hat{\mathbf{y}}1 - \hat{\mathbf{x}}1}{\sqrt{1+1}} = \frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}}.$$

In medium 1, normal component is

$$B_{1n} = \hat{\mathbf{n}}_2 \cdot \mathbf{B}_1 = \left( \frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}} \right) \cdot (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3) = \frac{3}{\sqrt{2}} - \frac{2}{\sqrt{2}} = \frac{1}{\sqrt{2}},$$

$$\mathbf{B}_{1n} = \hat{\mathbf{n}}_2 B_{1n} = \left( \frac{\hat{\mathbf{y}}}{\sqrt{2}} - \frac{\hat{\mathbf{x}}}{\sqrt{2}} \right) \cdot \frac{1}{\sqrt{2}} = \frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2}.$$

Tangential component is

$$\mathbf{B}_{1t} = \mathbf{B}_1 - \mathbf{B}_{1n} = (\hat{\mathbf{x}}2 + \hat{\mathbf{y}}3) - \left( \frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2} \right) = \hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5.$$

Boundary conditions:

$$B_{1n} = B_{2n}, \quad \text{or} \quad \mathbf{B}_{2n} = \frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2},$$

$$H_{1t} = H_{2t}, \quad \text{or} \quad \frac{B_{2t}}{\mu_2} = \frac{B_{1t}}{\mu_1}.$$

Hence,

$$\mathbf{B}_{2t} = \frac{\mu_2}{\mu_1} \mathbf{B}_{1t} = \frac{\mu_2}{\mu_1} (\hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5).$$

Finally,

$$\mathbf{B}_2 = \mathbf{B}_{2n} + \mathbf{B}_{2t} = \left( \frac{\hat{\mathbf{y}}}{2} - \frac{\hat{\mathbf{x}}}{2} \right) + \frac{\mu_2}{\mu_1} (\hat{\mathbf{x}}2.5 + \hat{\mathbf{y}}2.5).$$

For  $\mu_1 = 5\mu_2$ ,

$$\mathbf{B}_2 = \hat{\mathbf{y}} \quad (\text{T}).$$


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