5.38 In terms of the dc current I, how much magnetic energy is stored in the insulating medium of a 6 m long, air-filled section of a coaxial transmission line, given that the radius of the inner conductor is 5 cm and the inner radius of the outer conductor is 10 cm?

Solution: From Eq. (5.99), the inductance per unit length of an air-filled coaxial cable is given by

$$L' = \frac{\mu_0}{2\pi} \ln\left(\frac{b}{a}\right) \qquad \text{(H/m)}.$$

Over a length of 6 m, the inductance is

$$L = 6L' = \frac{6 \times 4\pi \times 10^{-7}}{2\pi} \ln\left(\frac{10}{5}\right) = 832 \times 10^{-9} \qquad \text{(H)}.$$

From Eq. (5.104), $W_\text{m} = LI^2/2 = 416I^2$ (nJ), where W_m is in nanojoules when I is in amperes. Alternatively, we can use Eq. (5.106) to compute W_m:

$$W_\text{m} = \frac{1}{2} \int_{\mathscr{V}} \mu_0 H^2 \, d\mathscr{V}.$$

From Eq. (5.97), $H = B/\mu_0 = I/2\pi r$, and

$$W_m = \frac{1}{2} \int_{z=0}^{6m} \int_{\phi=0}^{2\pi} \int_{r=a}^{b} \mu_0 \left(\frac{I}{2\pi r}\right)^2 r \, dr \, d\phi \, dz = 416I^2 \qquad \text{(nJ)}.$$