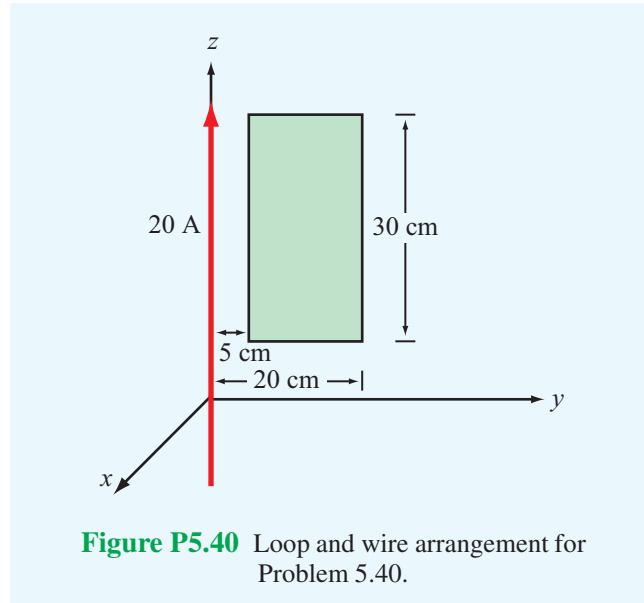


5.40 The rectangular loop shown in Fig. P5.40 is coplanar with the long, straight wire carrying the current $I = 20$ A. Determine the magnetic flux through the loop.



Solution: The field due to the long wire is, from Eq. (5.30),

$$\mathbf{B} = \hat{\phi} \frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi r} = -\hat{\mathbf{x}} \frac{\mu_0 I}{2\pi y},$$

where in the plane of the loop, $\hat{\phi}$ becomes $-\hat{\mathbf{x}}$ and r becomes y .

The flux through the loop is along $-\hat{\mathbf{x}}$, and the magnitude of the flux is

$$\begin{aligned} \Phi &= \int_S \mathbf{B} \cdot d\mathbf{s} = \frac{\mu_0 I}{2\pi} \int_{5 \text{ cm}}^{20 \text{ cm}} -\frac{\hat{\mathbf{x}}}{y} \cdot -\hat{\mathbf{x}} (30 \text{ cm} \times dy) \\ &= \frac{\mu_0 I}{2\pi} \times 0.3 \int_{0.05}^{0.2} \frac{dy}{y} \\ &= \frac{0.3 \mu_0}{2\pi} \times 20 \times \ln\left(\frac{0.2}{0.05}\right) = 1.66 \times 10^{-6} \quad (\text{Wb}). \end{aligned}$$
