5.6 A 20-turn rectangular coil with sides $l = 30$ cm and $w = 10$ cm is placed in the y–z plane as shown in Fig. P5.6.

(a) If the coil, which carries a current $I = 10$ A, is in the presence of a magnetic flux density

$$\mathbf{B} = 2 \times 10^{-2}(\hat{\mathbf{x}} + \hat{\mathbf{y}}2) \qquad \text{(T)},$$

determine the torque acting on the coil.

(b) At what angle ϕ is the torque zero?

(c) At what angle ϕ is the torque maximum? Determine its value.

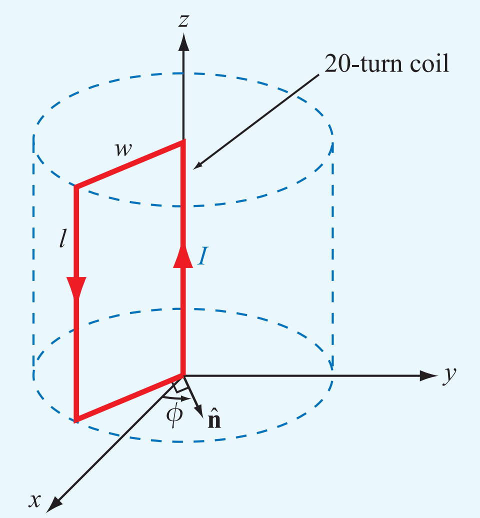

Figure P5.6 Rectangular loop of Problem 5.6.

Solution:

(a) The magnetic field is in direction $(\hat{\mathbf{x}} + \hat{\mathbf{y}}2)$, which makes an angle $\phi_0 = \tan^{-1}\frac{2}{1} = 63.43°$.

The magnetic moment of the loop is

$$\mathbf{m} = \hat{\mathbf{n}}NIA = \hat{\mathbf{n}}20 \times 10 \times (30 \times 10) \times 10^{-4} = \hat{\mathbf{n}}6 \qquad (\text{A}\cdot\text{m}^2),$$

where $\hat{\mathbf{n}}$ is the surface normal in accordance with the right-hand rule. When the loop is in the negative-y of the y–z plane, $\hat{\mathbf{n}}$ is equal to $\hat{\mathbf{x}}$, but when the plane of the loop is moved to an angle ϕ, $\hat{\mathbf{n}}$ becomes

$$\hat{\mathbf{n}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi,$$

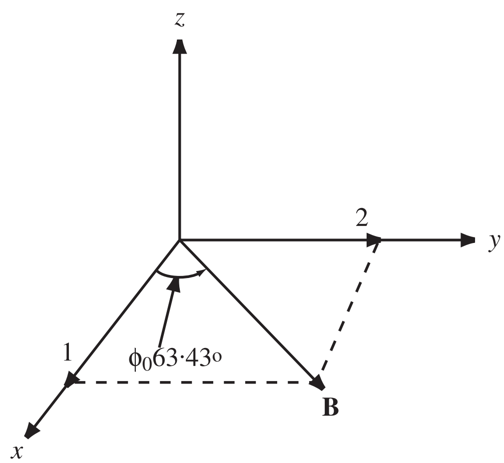

Figure P5.6 (a) Direction of **B**.

$$\begin{aligned}\mathbf{T} &= \mathbf{m}\times\mathbf{B} = \hat{\mathbf{n}}6\times 2\times 10^{-2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}2)\\ &= (\hat{\mathbf{x}}\cos\phi+\hat{\mathbf{y}}\sin\phi)6\times 2\times 10^{-2}(\hat{\mathbf{x}}+\hat{\mathbf{y}}2)\\ &= \hat{\mathbf{z}}0.12[2\cos\phi-\sin\phi] \qquad (\text{N·m}).\end{aligned}$$

(b) The torque is zero when

$$2\cos\phi-\sin\phi=0,$$

or

$$\tan\phi=2, \qquad \phi=63.43^\circ \text{ or } -116.57^\circ.$$

Thus, when $\hat{\mathbf{n}}$ is parallel to $\mathbf{B}$, $\mathbf{T}=0$.

(c) The torque is a maximum when $\hat{\mathbf{n}}$ is perpendicular to $\mathbf{B}$, which occurs at

$$\phi=63.43\pm 90^\circ=-26.57^\circ \text{ or } +153.43^\circ.$$

Mathematically, we can obtain the same result by taking the derivative of $\mathbf{T}$ and equating it to zero to find the values of ϕ at which $|\mathbf{T}|$ is a maximum. Thus,

$$\frac{\partial T}{\partial\phi}=\frac{\partial}{\partial\phi}(0.12(2\cos\phi-\sin\phi))=0$$

or

$$-2\sin\phi+\cos\phi=0,$$

which gives $\tan\phi=-\frac{1}{2}$, or

$$\phi=-26.57^\circ \text{ or } 153.43^\circ,$$

at which $\mathbf{T} = \hat{\mathbf{z}}0.27$ (N·m).