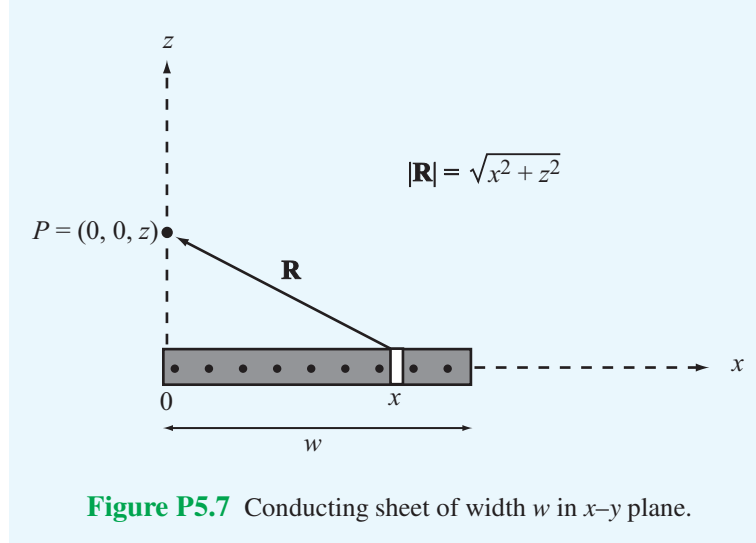


**5.7** An infinitely long, thin conducting sheet defined over the space  $0 \leq x \leq w$  and  $-\infty \leq y \leq \infty$  is carrying a current with a uniform surface current density  $\mathbf{J}_s = \hat{\mathbf{y}}5$  (A/m). Obtain an expression for the magnetic field at point  $P = (0, 0, z)$  in Cartesian coordinates.

**Solution:**



The sheet can be considered to be a large number of infinitely long but narrow wires each  $dx$  wide lying next to each other, with each carrying a current  $I_x = J_s dx$ . The wire at a distance  $x$  from the origin is at a distance vector  $\mathbf{R}$  from point  $P$ , with

$$\mathbf{R} = -\hat{\mathbf{x}}x + \hat{\mathbf{z}}z.$$

Equation (5.30) provides an expression for the magnetic field due to an infinitely long wire carrying a current  $I$  as

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} = \frac{\hat{\boldsymbol{\phi}}I}{2\pi r}.$$

We now need to adapt this expression to the present situation by replacing  $I$  with  $I_x = J_s dx$ , replacing  $r$  with  $R = (x^2 + z^2)^{1/2}$ , as shown in Fig. P5.10, and by assigning the proper direction for the magnetic field. From the Biot–Savart law, the direction of  $\mathbf{H}$  is governed by  $\mathbf{l} \times \mathbf{R}$ , where  $\mathbf{l}$  is the direction of current flow. In the present case,  $\mathbf{l}$  is in the  $\hat{\mathbf{y}}$  direction. Hence, the direction of the field is

$$\frac{\mathbf{l} \times \mathbf{R}}{|\mathbf{l} \times \mathbf{R}|} = \frac{\hat{\mathbf{y}} \times (-\hat{\mathbf{x}}x + \hat{\mathbf{z}}z)}{|\hat{\mathbf{y}} \times (-\hat{\mathbf{x}}x + \hat{\mathbf{z}}z)|} = \frac{\hat{\mathbf{x}}z + \hat{\mathbf{z}}x}{(x^2 + z^2)^{1/2}}.$$

Therefore, the field  $d\mathbf{H}$  due to the current  $I_x$  is

$$d\mathbf{H} = \frac{\hat{\mathbf{x}}z + \hat{\mathbf{z}}x}{(x^2 + z^2)^{1/2}} \frac{I_x}{2\pi R} = \frac{(\hat{\mathbf{x}}z + \hat{\mathbf{z}}x)J_s dx}{2\pi(x^2 + z^2)},$$

and the total field is

$$\begin{aligned}\mathbf{H}(0,0,z) &= \int_{x=0}^w (\hat{\mathbf{x}}z + \hat{\mathbf{z}}x) \frac{J_s dx}{2\pi(x^2 + z^2)} \\ &= \frac{J_s}{2\pi} \int_{x=0}^w (\hat{\mathbf{x}}z + \hat{\mathbf{z}}x) \frac{dx}{x^2 + z^2} \\ &= \frac{J_s}{2\pi} \left( \hat{\mathbf{x}}z \int_{x=0}^w \frac{dx}{x^2 + z^2} + \hat{\mathbf{z}} \int_{x=0}^w \frac{x dx}{x^2 + z^2} \right) \\ &= \frac{J_s}{2\pi} \left( \hat{\mathbf{x}}z \left( \frac{1}{z} \tan^{-1} \left( \frac{x}{z} \right) \right) \Big|_{x=0}^w + \hat{\mathbf{z}} \left( \frac{1}{2} \ln(x^2 + z^2) \right) \Big|_{x=0}^w \right) \\ &= \frac{5}{2\pi} \left[ \hat{\mathbf{x}}2\pi \tan^{-1} \left( \frac{w}{z} \right) + \hat{\mathbf{z}} \frac{1}{2} (\ln(w^2 + z^2) - \ln(0 + z^2)) \right] \quad \text{for } z \neq 0, \\ &= \frac{5}{2\pi} \left[ \hat{\mathbf{x}}2\pi \tan^{-1} \left( \frac{w}{z} \right) + \hat{\mathbf{z}} \frac{1}{2} \ln \left( \frac{w^2 + z^2}{z^2} \right) \right] \quad (\text{A/m}) \quad \text{for } z \neq 0.\end{aligned}$$

An alternative approach is to employ Eq. (5.24a) directly.

---