6.10 A 50-cm-long metal rod rotates about the z-axis at 90 revolutions per minute, with end 1 fixed at the origin as shown in Fig. P6.10. Determine the induced emf V_{12} if $\mathbf{B} = \hat{\mathbf{z}}\, 2 \times 10^{-4}$ T.

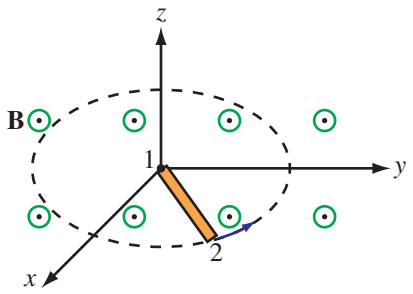

Figure P6.10 Rotating rod of Problem 6.10.

Solution: Since $\vec{B}$ is constant, $V_{\text{emf}} = V_{\text{emf}}^{\text{m}}$. The velocity $\vec{u}$ for any point on the bar is given by $\vec{u} = \hat{\boldsymbol{\phi}} r\omega$, where

$$\omega = \frac{2\pi \text{ rad/cycle} \times (90 \text{ cycles/min})}{(60 \text{ s/min})} = 3\pi \text{ rad/s}.$$

From Eq. (6.24),

$$\begin{aligned} V_{12} = V_{\text{emf}}^{\text{m}} = \int_2^1 \left(\vec{u} \times \vec{B}\right) \cdot d\vec{l} &= \int_{r=0.5}^{0} \left(\hat{\boldsymbol{\phi}} 3\pi r \times \hat{\mathbf{z}} 2 \times 10^{-4}\right) \cdot \hat{\mathbf{r}}\, dr \\ &= 6\pi \times 10^{-4} \int_{r=0.5}^{0} r\, dr \\ &= 3\pi \times 10^{-4} r^2 \Big|_{0.5}^{0} \\ &= -3\pi \times 10^{-4} \times 0.25 = -236 \quad (\mu\text{V}). \end{aligned}$$