6.11 The loop shown in P6.11 moves away from a wire carrying a current $I_1 = 10$ A at a constant velocity $\mathbf{u} = \hat{\mathbf{y}}7.5$ (m/s). If $R = 10\ \Omega$ and the direction of I_2 is as defined in the figure, find I_2 as a function of y_0, the distance between the wire and the loop. Ignore the internal resistance of the loop.

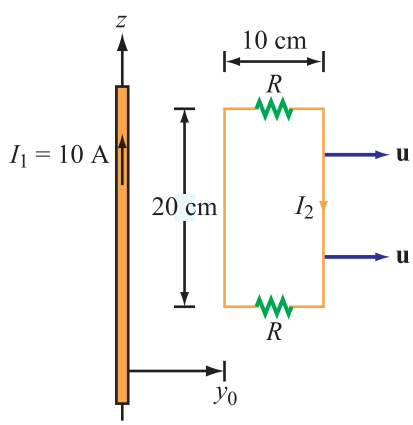

Figure P6.11 Moving loop of Problem 6.11.

Solution: Assume that the wire carrying current I_1 is in the same plane as the loop. The two identical resistors are in series, so $I_2 = V_{\text{emf}}/2R$, where the induced voltage is due to motion of the loop and is given by Eq. (6.26):

$$V_{\text{emf}} = V_{\text{emf}}^{\text{m}} = \oint_C \left(\vec{u} \times \vec{B}\right) \cdot d\vec{l}.$$

The magnetic field $\vec{B}$ is created by the wire carrying I_1. Choosing $\hat{\mathbf{z}}$ to coincide with the direction of I_1, Eq. (5.30) gives the external magnetic field of a long wire to be

$$\vec{B} = \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r}\ .$$

For positive values of y_0 in the y-z plane, $\hat{\mathbf{y}} = \hat{\mathbf{r}}$, so

$$\vec{u} \times \vec{B} = \hat{\mathbf{y}}|\vec{u}| \times \vec{B} = \hat{\mathbf{r}}|\vec{u}| \times \hat{\boldsymbol{\phi}} \frac{\mu_0 I_1}{2\pi r} = \hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r}\ .$$

Integrating around the four sides of the loop with $d\vec{l} = \hat{\mathbf{z}}\, dz$ and the limits of integration chosen in accordance with the assumed direction of I_2, and recognizing that only the two sides without the resistors contribute to $V_{\text{emf}}^{\text{m}}$, we have

$$V_{\text{emf}}^{\text{m}} = \int_0^{0.2} \left(\hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r}\right)\bigg|_{r=y_0} \cdot (\hat{\mathbf{z}}\, dz) + \int_{0.2}^{0} \left(\hat{\mathbf{z}} \frac{\mu_0 I_1 u}{2\pi r}\right)\bigg|_{r=y_0+0.1} \cdot (\hat{\mathbf{z}}\, dz)$$

$$= \frac{4\pi \times 10^{-7} \times 10 \times 7.5 \times 0.2}{2\pi} \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right)$$

$$= 3 \times 10^{-6} \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right) \qquad \text{(V)},$$

and therefore

$$I_2 = \frac{V_{\text{emf}}^{\text{m}}}{2R} = 150 \left(\frac{1}{y_0} - \frac{1}{y_0 + 0.1} \right) \qquad \text{(nA)}.$$