6.19 At $t = 0$, charge density ρ_{v0} was introduced into the interior of a material with a relative permittivity $\varepsilon_r = 6$. If at $t = 1$ μs the charge density has dissipated down to $10^{-3}\rho_{v0}$, what is the conductivity of the material?

Solution: We start by using Eq. (6.61) to find τ_r:

$$\rho_v(t) = \rho_{v0} e^{-t/\tau_r},$$

or

$$10^{-3}\rho_{v0} = \rho_{v0} e^{-10^{-6}/\tau_r},$$

which gives

$$\ln 10^{-3} = -\frac{10^{-6}}{\tau_r},$$

or

$$\tau_r = -\frac{10^{-6}}{\ln 10^{-3}} = 1.45 \times 10^{-7} \qquad \text{(s)}.$$

But $\tau_r = \varepsilon/\sigma = 6\varepsilon_0/\sigma$. Hence

$$\sigma = \frac{6\varepsilon_0}{\tau_r} = \frac{6 \times 8.854 \times 10^{-12}}{1.45 \times 10^{-7}} = 3.66 \times 10^{-4} \qquad \text{(S/m)}.$$