

6.22 In a certain medium, the direction of current density \mathbf{J} points in the radial direction in cylindrical coordinates and its magnitude is independent of both ϕ and z . Determine \mathbf{J} , given that the charge density in the medium is

$$\rho_v = \rho_0 r \cos \omega t \quad (\text{C/m}^3).$$

Solution: Based on the given information,

$$\mathbf{J} = \hat{\mathbf{r}} J_r(r).$$

With $J_\phi = J_z = 0$, in cylindrical coordinates the divergence is given by

$$\nabla \cdot \mathbf{J} = \frac{1}{r} \frac{\partial}{\partial r} (r J_r).$$

From Eq. (6.54),

$$\nabla \cdot \mathbf{J} = -\frac{\partial \rho_v}{\partial t} = -\frac{\partial}{\partial t} (\rho_0 r \cos \omega t) = \rho_0 r \omega \sin \omega t.$$

Hence

$$\begin{aligned} \frac{1}{r} \frac{\partial}{\partial r} (r J_r) &= \rho_0 r \omega \sin \omega t, \\ \frac{\partial}{\partial r} (r J_r) &= \rho_0 r^2 \omega \sin \omega t, \\ \int_0^r \frac{\partial}{\partial r} (r J_r) dr &= \rho_0 \omega \sin \omega t \int_0^r r^2 dr, \\ r J_r \Big|_0^r &= (\rho_0 \omega \sin \omega t) \frac{r^3}{3} \Big|_0^r, \\ J_r &= \frac{\rho_0 \omega r^2}{3} \sin \omega t, \end{aligned}$$

and

$$\mathbf{J} = \hat{\mathbf{r}} J_r = \hat{\mathbf{r}} \frac{\rho_0 \omega r^2}{3} \sin \omega t \quad (\text{A/m}^2).$$
