6.23 The electric field of an electromagnetic wave propagating in air is given by

$$\mathbf{E}(z,t) = \hat{\mathbf{x}}4\cos(6\times 10^8 t - 2z) + \hat{\mathbf{y}}3\sin(6\times 10^8 t - 2z) \qquad \text{(V/m)}.$$

Find the associated magnetic field $\mathbf{H}(z,t)$.

Solution: Converting to phasor form, the electric field is given by

$$\tilde{\vec{E}}(z) = \hat{\mathbf{x}}4e^{-j2z} - j\hat{\mathbf{y}}3e^{-j2z} \qquad \text{(V/m)},$$

which can be used with Eq. (6.87) to find the magnetic field:

$$\begin{aligned}
\tilde{\vec{H}}(z) &= \frac{1}{-j\omega\mu}\nabla\times\tilde{\vec{E}} \\
&= \frac{1}{-j\omega\mu}\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 4e^{-j2z} & -j3e^{-j2z} & 0 \end{vmatrix} \\
&= \frac{1}{-j\omega\mu}(\hat{\mathbf{x}}6e^{-j2z} - \hat{\mathbf{y}}\,j8e^{-j2z}) \\
&= \frac{j}{6\times 10^8\times 4\pi\times 10^{-7}}(\hat{\mathbf{x}}6 - \hat{\mathbf{y}}\,j8)e^{-j2z} = j\hat{\mathbf{x}}8.0\exp - j2z + \hat{\mathbf{y}}10.6\exp - j2z \qquad \text{(mA/m)}.
\end{aligned}$$

Converting back to instantaneous values, this is

$$\vec{H}(t,z) = -\hat{\mathbf{x}}8.0\sin\left(6\times 10^8 t - 2z\right) + \hat{\mathbf{y}}10.6\cos\left(6\times 10^8 t - 2z\right) \qquad \text{(mA/m)}.$$